AISB 2011

Social Networks and Multiagent Systems

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THE UNIVERSITY *of York*

Foreword from the Convention Chairs

The AISB'11 call for symposium proposals particularly encouraged events drawing more strongly on the cognitive science aspect of the AISB remit. The result is a coherent programme with a very strong interdisciplinary character, which is also matched in the choice of plenary speakers. The three symposia looking at the interaction between Computing and Philosophy, the prospect of machine consciousness and the quest for a new, comprehensive intelligence test, form a coherent unit where the eternal questions of who we are and what makes us so are asked from a dual Human-Machine perspective. The Symposia on Active Vision, Computational Models of Cognitive Development and Human Memory for Artificial Agents demonstrate how better understanding of the nature and basis of cognitive processes can advance work on Artificial Intelligence and, inversely, how computational models of these processes can help better to understand them. The prominent multi-agent design and modelling paradigm links the Symposium on Social Networks and Multi-agent Systems with the one on AI and Games. Finally, the Symposium on Learning Language Models from Multilingual Corpora, which brings together some of the first attempts in this area, can also be seen through the prism of such a general notion in Philosophy and Linguistics as semiosis, and the dual role of sign and interpretant that text plays in translations.

We are delighted that after another ten successful years in its long history, the AISB convention is returning to the University of York. The 2011 convention takes place on the brand-new Heslington East campus, the result of a multi-million pound expansion that is now the new home of the Department of Computer Science, and hosts the Excellence Hub for Yorkshire and Humber, a new incubator for interdisciplinary research and interaction between academia and industry. The last few years have seen a strong involvement of the Computer Science Department in such interdisciplinary collaboration through the York Centre for Complex Systems Analysis (YCCSA), and we hope that this convention will provide a boost for more synergy between York departments, with other institutions conducting AI-related research in the region, and beyond. As the programme shows, we have also made an effort to promote cooperation with industry and use the convention to support school outreach. The convention format makes it perfect for establishing dialogue and collaboration in new areas of research, as well as across disciplines, and we hope that this year, it will play again this role to the full. We want to thank everyone who has contributed to it or otherwise made this event possible and wish all participants a fruitful and enjoyable time in York.

Dimitar Kazakov and George Tsoulas

Social Networks And MultiAgent Systems

3rd Symposium

http://snamas2011.res-ear.ch

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Analysis of Power Networks among the Actors of a Social Organization

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Abstract. The Sociology of the Organized Action studies how social organizations are regularized, due to the balancing process between the power relationships of the social actors. A formalization of this theory allows drawing power social networks of an organization. On the other hand, the Social Network Analysis has developed efficient tools to improve the understanding of the structure and behavior of social networks. The paper shows how these tools are fruitfully applied to power networks.[12]

1 INTRODUCTION

The Sociology of the Organized Action [3][4][8] studies how social organizations (for example a firm) are regularized as a result of the balancing process among the power relationships between the social actors. A formalization of this theory allows designing models of social organizations and to simulate the "social game" [17][13][16]. Therefore, it is possible to draw various power networks among the actors of a social organization, from the potential power of the actors or from their effective power, in a specific state of the organization.

On the other hand, Social Network Analysis (SNA) studies the structure of relations between actors who are tied by some kind of social relationship (like friendship, collaboration, counsel and so on) [20]. In this context, [10] analyses the structure of relations inside organizations to infer power relationships between the actors from their positions (see also [15]).

The purpose of this paper is to start directly from power networks of a social organization designed from the SOA formalization, and to study how the tools proposed by SNA can improve their understanding. Especially, the centrality metric allows investigating properties of the power like its intensity, its diversity and its scope.

In section 2, we briefly present the Sociology of the Organized Action and how its formalization enables to provide a formal definition of the power of an actor. Then, we recall the definitions of some concepts of the Social Network Analysis (section 3) and we introduce, in section 4, the different power networks that can be drawn from the SAO formal model of an organization. In section 5, we propose an analysis of the power networks with the SNA's tools.

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2 A FORMALIZATION OF THE SOCIOLOGY OF THE ORGANIZED ACTION

2.1 A brief introduction to the Sociology of the Organized Action

The Sociology of the Organized Action (SOA) [3][4][8] is a well-experienced theory, acknowledged by more recent theories of organizations [5]. It aims at discovering and explaining the real functioning of social organizations, or so called Concrete Systems of Action (CSA), beyond their formal rules (in the form of organization charts, directive instructions, position descriptions, protocols and so on), and it focuses on the regularization phenomenon which ensures their relative synchronic stability.

The SOA assumes that an organization is a social construct that is both the produce and the shape of the interactions among its member actors. Moreover, it assumes that each actor behaves strategically although he has only bounded rationality capabilities [19]. Therefore the behavior of each actor is neither totally conditioned by the organizational rules that constrain him, nor induced by purely individual factors. And it is strategic as it intends to realize the actors' aims, or goals, would they be conscious or not.

To get the means to achieve his goals, every actor seeks to have enough power to be able to preserve or increase his autonomy and acting capacity within the organization. According to the SOA, his power results from the mastering of one or several uncertainty zones (UZ) that enable him to behave in a way that is unpredictable for other actors and consequently to set, to some extent, the exchange rules in his relations with others. The UZs are the media of the power relationships between the actors; each UZ involves one (or several) resource needed for the action, and thus it is both a constraint and an opportunity. Each social actor both controls some UZs and depends on some others, so that they are reciprocally [18] dependent on each other.

2.2 The meta-model to describe the structure of an organization

Purposing to enable the design of models of social organizations, we rely on a meta-model as an abstraction that catches the constitutive concepts and properties of social organizations [16] [13]; it can be instantiated on specific cases as models of concrete organizations.

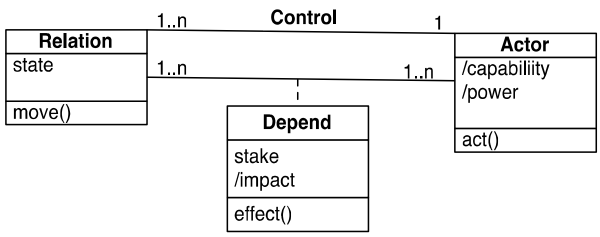

Figure 1. The meta-model of the structure of organizations

The meta-model is shown in figure 1, as a UML class diagram. It is composed of a class *Actor* and a class *Relation*, linked by the associations *Control* and *Depend*. Actors are the active entities who handle the relations. Each relation is controlled by a unique actor, the only one able to change its *state*. Each actor distributes his *stakes* over the relations he is dependent on. Thus, every actor gets some *impact* from each of these relations, given by the *effect()* function applied to the state of the relation, weighted by the stake of the actor. Consequently, he gets some *capability*, or *action capacity*, as an aggregation of the impacts he receives from all the relations he depends on. He also exerts some *power*, as an aggregation of capacities he grants to the actors who depend on the relations he controls.

These elements will now be described in more details (see also [16]).

Resources and Relations

Every relation is founded on an organization's resource, or a set of resources in conjunction with one another (see figure 2), and it is controlled by a single actor. Resources are physical or cognitive elements required to achieve some actions, so that their availability is necessary for some actors. A relation refers to a certain type of recurrent interactions among actors about a specific use of the resources it is based on. Thus, the controller of a relation, who is in a position to determine how the resources are available to the others, controls to what extent the dependent actors are able to reach their goals.

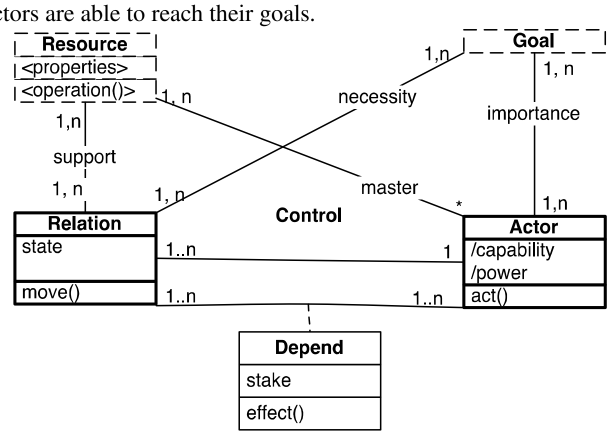

Figure 2. A representation of the resources and the goals, behind the relations and the actors' stakes of the meta-model[3]

The state attribute of a relation represents how the supporting resources are managed by its controller, that is the behavior of the controller actor with regard to these resources. It is defined inside a space of behavior SB, representing the set of behaviors that the actor can choose to manage the relation. This space of behavior is represented for any relation as the interval SB = [-1;1]. This interval is bipolar: -1 represents the least cooperative behaviors of the controller, 1 represents the most cooperative behaviors, while the zero value stands for neutral behaviors.

The actor who controls a relation may change his behavior by using the function *move()* that changes the value of the state of the relation. An actor modifies his behavior towards a greater or lower cooperation with regard to his previous behavior.

The state of a relation determines the availability of the underlying resources, i.e., how each dependent actor is granted to use them according to his needs. So, the state of a relation produces an effect on every depending actor: his capacity to use the resources as he would like. Therefore, the greater the effect for a given actor, the more useable the resource, and the larger his capacity of action to reach his goals. Effects take values on an arbitrary scale from -10 to 10:

worst access = -10, ..., neutral = 0, ..., optimal access =10.

Effect values are given by an *effect()* function, which is defined depending on the nature of the relation. The $effect_r()$ function of a relation *r* is defined as:

$effect_r : A \times SB_r \dashrightarrow [-10, 10]$

where A stands for the set of actors, SB_r for the space of behavior of the relation *r*, and [-10, 10] is the range of action capacity.

Actors and their Stakes

Actors and resources are defined in relation one to another, in a dialogical way: something is a resource if and only if some actors depend on it, and someone is an actor if and only if he masters some needed resources, and thus some relations. The SOA assumes that actors are strategic: they have goals that lead them to find some ways to reach them. According to the relative importance of his goals and the necessity of resources to reach these goals (see figure 2), each actor distributes stakes on the relations; we assume that each actor has 10 stake marks to distribute over the relations. The more valuable a resource for an actor, the higher his stake on the supported relation. Stakes are represented by numerical coefficient, on an arbitrary scale:

null = 0, negligible = 1,… ,significant = 5,… , critical = 10.

Capability and Power of Actors

Defining the state of an organization as the vector of every relation state, each state of an organization allows considering the aggregation of the effects of the relations on an actor, over all the relations he depends on, weighted by the stake he puts on each relation. Such a quantity depicts the overall ability for an actor to gain access to the resources he needs to reach his goals, weighted by the relative importance of these resources regarding his goals. It measures, for an actor, his action capacity.

Under the hypothesis that there are no interferences between the use of resources, when the organization is in the state s = (s_{r1}, ..., s_{rn}), this action capacity is defined as:

$$action\_capacity(a, s) = \sum_{r \in R} impact_r(a, s) \qquad (1)$$

where

$$impact_r(a, s) = stake(a, r) * effect_r(a, s) \qquad (2)$$

By controlling some relations, every actor contributes to the action capacity of the actors who depend on these relations, i.e. to the access of the resources they need. The influence of an actor on the action capacity of another one, i.e. how much action capacity he gives to him, fits the concept of power, a core

[3] Resources and goals are implicit, not in the meta-model

concept in the SOA. The power exerted by an actor *a* on an actor *b* in a state *s* of the organization, is defined as:

$$power(a,b,s) = \sum_{r \in R;\ a\ controls\ r} |impact_r(b, s_r)|, \quad (3)$$

the global power of an actor a is defined as:

$$power(a,s) = \sum_{b \in A} power(a, b, s)^4. \quad (4)$$

and the total amount of power within the organization as:

$$power(s) = \sum_{b,\ a \in A} power(a, b, s) \quad (4')$$

The meta-model is completed with a couple of elements (e. g. constraints between the states of relations and solidarities between actors) that are not necessary to consider in this paper (see [16] for more details).

2.3 The social game of an organization

According to the meta-model, the model of an organization is a structure including:

- A, the (finite and non-empty) set of actors;
- R, the (finite and non-empty) set of relations;
- control : R → A;
- state : R → SB_r = [-1, 1];
- stake : A x R → [0, 10] such as $\forall a \in A, \sum_{r \in R} stake\ (a, r) = 10$, and an actor a is said to be dependent on a relation r iff stake(a, r) > 0;
- effect: R x A x [-1, 1] → [-10, 10].

Such a structure defines a "social game" where each actor looks for having a satisfying level of capability. To realize this meta-goal, each actor adjusts his behavior by modifying the state of the relations he controls. By this way, he modifies the capability of the actors who depend on these relations and, indirectly, his own capability. The game is over when a stationary state is reached: the organization is in a sustainable state, since the actors no longer look for modifying the state of the relations because each one is satisfied by his level of capability.

2.4 The SocLab software environment

The *SocLab* environment allows editing (the model of) the structure of an organization compliant with the meta-model, to display the value of relevant indicators (action capacity, power, see also section 4), to explore the states of the organization (e.g. the ones that maximize or minimize the capacity or the power of an actor, the Nash equilibriums, …). Implementing the model of an organization as a MultiAgents System (where the agents represent the actors of the model) and giving to the agents a behaviour model, SocLab allows the actors to play the "social game" and to reach socially feasible states of the organization, by simulation [13]. See [6] for more details about the representation of the agents and the simulation algorithm.

[1] reports the use of SocLab for the assessment of the social acceptability of new agricultural policies in the upstream part of the Gers river's basin.

[4] We may also distinguish the *positive power* that retains only the positive impacts from the *negative power*.

3 METRICS IN SOCIAL NETWORK ANALYSIS

We purpose in this paper to relate the model of an organization presented in section 2 to tools proposed by the Social Network Analysis. Therefore, we present in this section this tools.

SNA describes social structures as networks of well specified ties that bind individuals or organizations one to another. Using concepts of network theory, it aims to reveal the context in which the interactions between the individuals occur, providing some insights about the effect of the structure on these interactions. Sexual relationships, friendship or financial transaction networks are some of the famous kinds of social networks having been studied (see [11][20]).

The nature of the social relationship which defines a social network is crucial: it determines the type of investigated network and the meaning of every metric used to characterize a node, an edge or the whole network.

The standard structure of networks in SNA is an undirected graph i.e. a graph where nodes are tied by a symmetric binary relation, as most of the relations types covered by SNA imply reciprocity in the course of interactions.

When the goal of a SNA study is to seize the power between the members of a social organization, the relations are restrained to more specific kinds of relationship, such as information communication, support or counsel taking relations. In these more specific networks, it is commonly admitted that power arises from occupying an advantageous position in the social relationship network [2].

The measure of the advantage provided by a particular position in a network uses three well-known metrics: the degree centrality, the closeness centrality and the betweenness centrality.

Degree centrality is defined as the number of ties a node is connected to. A node with a high degree centrality is a node which is directly connected to many nodes of the network, so that the corresponding individual has many opportunities to interact with others.

Closeness centrality refers to the distance between a node and the others. A node with a high value of closeness centrality is a node which is relatively near of the other nodes [7].

$$C_{closeness}(\alpha) = \frac{1}{\sum_{\beta \in V} d(\alpha, \beta)}$$

where $d(\alpha, \beta)$ stands for the (geodesic) distance between vertices α and β. A high degree centrality of an individual also relates to his possibility to interact with many others, even in an indirect way.

Betweenness centrality addresses the occurrence of a node on the (shortest) paths that link the other nodes to each other [20].

$$C_{betweenness}(\alpha) = \sum_{\alpha \neq \beta \neq \gamma \in V} \left(\frac{\sigma_{\beta\gamma}(\alpha)}{\sigma_{\beta\gamma}}\right)$$

where α, β and γ are vertices of the network, $\sigma_{\beta\gamma}$ stands for the shortest paths from β to γ that contains α.

A node with a high value of betweenness centrality has the possibility to be involved in many transactions that flow in the network.

The combination of these three metrics allows a node to be characterized regarding its importance in the network. This importance comes both from the number of nodes it can affect (its mean of action) and from the distance that separates it from the others (its action's range). As obvious as it may seem, the centralities scores that rely on the distance between two nodes only make sense if the length of a path between these two nodes is relevant regarding the nature of the relation. It requires that the relations should somehow be *transitive* in order for a node to affect another one which it is not directly connected to.

Another metric currently used in SNA is the *structural equivalence* that reflects, for a pair of nodes, to what extent they are connected to the same nodes of the network. Two nodes said to be structurally equivalent share the same set of constraints and opportunities in the network.

Since having two nodes in a situation of exact structural equivalence is very rare, SNA commonly turns to numerical methods to measure the degree of structural equivalence that is, the similarity of nodes connections profiles [9].

4 FROM ORGANIZATION MODELS TO NETWORKS OF POWER

In section 2, we have presented the meta-model whereby we construct organization models. An organization model describes explicitly the structure of the set of relations that ties the actors one to another. Several networks of actors can be extracted from this structure. Since every actor controls at least one relation in the organization and depends on at least one relation he does not control, these networks are always strongly connected.

The first kind of network to be extracted from an organization model is an *actor-relation digraph*. An arc *(a, r)* represents a control relation from an actor *a* to a relation *r*, and an arc *(r, a)* represents a *dependency* from an actor *a* on a relation *r.*

The other kinds of networks to be extracted are directed *actor graphs*, in which an arc *(a, b)* represents a control-dependency relation i.e. *a* controls a relation on which *b* depends. As the SNA deals with actor graphs, we mainly focus on this second kind of networks in this paper.

These networks can be weighted with various measures that are either related to a specific state of the organization (*contextual actor network)* or only depend on the structure of the organization (*structural actor network).*

The contextual actor networks reflect a particular state of the organization. An arc *(a, b)* may be weighted by the aggregated impacts that *b* receives from the relations controlled by *a*, that is *power(a, b, s).* This network expresses the power that *a* exercises on *b*. In the (frequent) case where both *power(a, b, s)* and *power(b, a,, s)* are not null, only the arc with the highest value may be retained, labelled with the (positive) value of their difference, expressing the *relative power* of actors one on another. Another way of doing is to label undirected arcs by the sum of the absolute value of powers, expressing the intensity of the interactions between two actors.

In such networks, the actors nodes can be labelled with various indicators, such as the *capability* or the *power* (see section 2.2), or the *autonomy* of an actor (his capability to prevent others to control the resources he needs)., computed in the given state. We define the *autonomy* of an actor *a* as the sum of every impact received from relations that *a* itself controls.

The structural actor networks reflect structural properties of the organization, i.e. properties that are not computed for a given state. The structures of these networks are the same as those of the contextual ones, but they are weighted with state-independent measures.

For example, a structural network can be labelled with the minimum or maximum possible values of a contextual network, or by the mean value or the differences between these values. This kind of network is used to get insights on the range of situations of dependence in which the actors could be.

In order to highlight the importance of some specific resources of the organizations, relations nodes may be weighted by their *relevance*, defined as the sum of the stakes put by the actors on them.

The Figure 3 shows an example of a structural actor-relation network where actors are displayed as circle nodes and relations as square nodes. Plain arrows reflect *control* and dotted arrows reflect *dependence* relation. The size of squares represents the relevance of the relations.

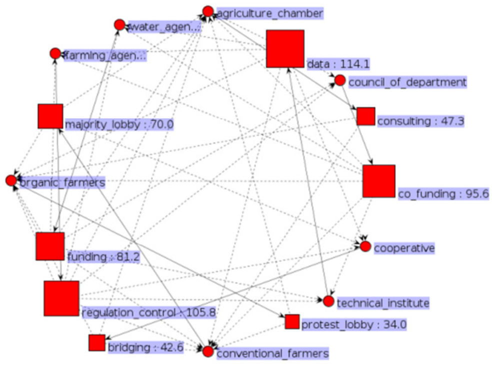

Figure 3. Actor-Relation Network showing the relative relevance of relations in an organization network (from SocLab)

In order to represent the amplitude of the effect a relation may have on an actor, arcs in actor-relation digraphs can be weighted by the *strength* of the relation on the actors who depend on it, defined as follows :

$$strength(r,a) = max \{effect_r(r,x)-effect_r(r,y)\} ; x,y \in SB_r$$

The same operation can be performed regarding the power of an actor: actors nodes might be labelled with the *structural power* indicator, that is the amplitude of its global power :

$$structural\ power(a,s)=max\{power(a,x)-power(a,y)\},\ x,y \in \textstyle\prod_r SB_r$$

Finally, following the SOA idea that, for an actor, power arises from the mastering of uncertainty zones that allows him to behave in an unpredictable way, representing the level of uncertainty in actor power networks would be useful to clarify power relationship in an organization.

One way of measuring this degree of uncertainty is to consider the standard deviation of relations states values, computed from the results of numerous simulations of the actors behavior regulation. Since these values reflect the diversity of behaviors that an actor could adopt concerning the handling of the resources he controls, weighting the arcs of a structural actor network with these values reflects the degree of uncertainty in the network.

5 ANALYSIS OF POWER NETWORKS

The theory of social networks has developed many tools to analyse properties of social networks (see section 3). According to the nature of the links that are considered, these properties enable to improve the understanding of the social structure that is described by the network. As power and dependency relationships are essential for the functioning of any organization, especially for the SOA theory, we propose in this section to analyse them with the tools of social networks to investigate properties of the power such as its *intensity,* its *diversity* or its *scope,*

The SocLab environment allows displaying tables that show the values of the various kinds of links between actors that have been presented in section 4. However, power relationships are not just a dyadic matter, because they spread among the actors and, most often, there are several actors that depend on one relation [14]. So, a network representation of these relationships enables to get a global view of the distribution of these relations, in addition to the local view focusing on each actor.

In the following, we will not consider the specificities of the different kinds of networks that have been introduced in the previous section; we just consider networks of power relationships (that we will call power networks).

In these networks, the arcs are labelled and directed, unlike social networks that are usually considered in SNA. The output arcs of a node are related to the power of the corresponding actor, while its input arcs are related to his action capacity

Distant power

The SNA make the assumption that the relations are transitive in order for a node to affect another one which is not directly connected to (see section 3). In the case of power networks, this means that we assume some transitivity of power relationships: if actor a exerts some power on actor b and the latter some power on actor c, then actor a exerts an indirect power on c. This view is compliant with the principles of the SAO regarding the behaviors of social actors: for the SAO, the actors “exchange their behaviors” the one with another, so that the behaviour of an actor (i.e. the power he exerts) depends on his capability (i.e. the power that he receives) [8].

How does the power of an actor a on another actor b propagates to a third actor c, depending on the power of b on c?

We can consider that the behavior of b depends on a, let us call it the *influence* of a on $b,$ to the extent of his dependency with regard to a, in other words the relative power that a exert on b, that is:

$$influence(a, b) = power(a, b)/capacity(b), \qquad (5)$$

so that:

$$power(a, c) = influence(a, b) * power(b, c). \qquad (6)$$

Moreover, if a controls two actors b_1 and b_2 who control c, then the power of a on c must be defined as:

$$power(a, c) = \sum_{1,2} influence(a, b_i) * power(b_i, c).$$

More generally, if we have a chain of power dependency a_0, a_1, …, a_n from an actor a_0 to an actor a_n, the distant power of a_0 on a_n is then evaluated as:

$$power_d(a_0, a_n)=(\prod_{0 \ldots n-1} influence(a_i, a_{i+1})) * power(a_{n-1}, a_n). \qquad (7)$$

Notice that this formula of distant power generalizes the definition of power given in 2.2 (3) in the case a and b are directly linked. The distant power exerted by an actor a on another actor b is given by the $(a,b)^{th}$ entry of the following matrix product:

$$Inf^{d-1} * P$$

Where Inf is the matrix of influence, whose $(i,j)^{th}$ entry, Inf_{ij}, is *influence(i,j)*, d is the length of the shortest path between nodes i and j and P is the matrix of (direct) power whose $(i,j)^{th}$ entry, P_{ij}, is *power(i,j)*.

Centrality of output arcs: intensity, diversity and scope of the power

In a power network, the *intensity* of the power of an actor, that is his amount of power, is given by the sum of the labels of the output arcs of the node.

The degree centrality of this node reported to the output arcs, i.e. the number of output arcs, indicates the *diversity* of the power, that is, the number of means that are at the corresponding actor’s disposal to exert his power. Having a high diversity power gives the actor the possibility to choose the most appropriate means to exert power according to the current situation, and thus to be well equipped to face various configurations of the organization.

The closeness centrality of a node (reported to the output arcs) is related to the *scope* of the corresponding actor’s power, that is how far and how many actors the actor is able to influence. Closeness centrality compute the length of the paths; to evaluate what can circulate along these paths, we consider the sum of the distant powers exerted by an actor on all the other ones:

$$\sum_{b \in A} power_d(a, b) \qquad \text{cf. (7),}$$

possibly divided by the total Power of the organization (4’) to get a normalized value.

Centrality of input arcs: distributed dependency and pressure

The input arcs of a node express both the action capacity received by the corresponding actor and his entailed dependence on other actors.

A high level of input degree centrality means that the actor does not depend on a single or few actors, but his dependency is distributed on several actors; thus, he can expect that in the case of the defection of some actor(s), others will compensate his loss, especially if they are in conflict with the defecting actor(s). As for the output arcs, the actor seems to be well equipped to face various configurations of the organization.

The closeness centrality of a node (reported to the input arcs) expresses the pressure that the actor has to endure from others and, symmetrically to the output arcs case, it may be evaluated by $\sum_{b \in A} power_d(b, a)$. However, we have to consider that a also exerts some distant influence (5) on b, since the network is strongly connected. The network representation of power relationships gives evidence of their looping nature, and this point deserves to be further investigated [14].

Actors’ structural position

In usual social networks, two actors who are structurally equivalent are in some way interchangeable, they feature no specificity and thus they are potentially not endowed with high power possibilities. In the case of power networks, the structural equivalence has to be examined distinguishing the similarities of output and inputs arcs. We propose the interpretation shown in table 1.

		output arcs	
		same	**different**
input arcs	**same**	coalition	solidarity
	different	conflict	

Table 1: interpretation of actors' structural equivalence

If two actors have the same input arcs and different output arcs, they depend on the same actors and have similar goals, at least goals that require the same capability. They are prone to influence the actors they depend on in the same direction, even to make the organization to evolve in the same direction. They show solidarity since their interests coincide, what is good for one of them does for the other one too.

Two actors that have the same output arcs but different input arcs exert the same power upon others but they have different goals. Thus they have divergent interests in the social game and it is likely that each one would like eliminate the other one in order to exert the totality of the power on the actors who depend on them. They are concurrent and each one would take benefice of a weakening of the other one.

Finally, two actors having the same input and output arcs are equivalent in the sense of SNA; they are redundant actors. In this situation, they would have advantage to constitute a coalition, if it is not already the case, to coordinate their action and jointly reinforce their power and their action capacity.

6 CONCLUSION

In this paper, we analyse power networks with tools of the Social Network Analysis. We construct these power networks from organization models elaborated with the SocLab meta-model which formalizes a well-established theory of the sociology of organization, the Sociology of the Organized Action. The meta-model defines structural and contextual measures which can be used to weight the nodes and the arcs of these networks.

The SNA, when it considers relations among the actors of an organization, intends to infer the power relationships between the actors from their positions in social networks, assuming that social actor are able to transform their structural advantage into a behavior that expresses their power [12]. Our proposition starts from networks that already describe the actors' power positions and thus investigate more finely properties of the power (for example its intensity or its diversity). However, the power networks constructed with the SAO and SocLab models are not the result of a direct observation like the SNA networks. Therefore, their accuracy is more questionable.

There are other concepts of SNA which deserve to be applied to power networks. For example, the diameter of a network (i.e. the length of the longest shortest paths between two nodes) can be interpreted as power centralization. Concerning the betweenness centrality, it would be interesting to distinguish different cases according to the autonomy level of the actors.

Finally, other measures can be used to weight power networks. It would be interesting to extend this work by using other measures and analysing these power networks.

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The “Logic” of Power.
Hints on How my Power Becomes his Power

Cristiano Castelfranchi [1]

Abstract. We analyze how the power of an agent creates social power over the other agents; how an agent acquires new powers, and a given power becomes a different power; how a power is transferred from one agent to another one and accumulated; how co-powers require coordination. What is power ‘alienation’ and ‘subjection’, and a power 'capital'.[2]

1 POWER: THE OTHER (DARK)SIDE OF DEPENDENCE

One of the most important effects of dependence is that it creates "power-over" [1]:

<1> *if Y depends on X (as for his goal Gy: that P) automatically X gets a “power-over” Y: the power of letting/making Y to realize/achieve his goal Gy: that P.*

X comes to have an "incentive/reward power"[3]: X can provide to Y the reward of the realization of P; and can "promise" this (incentive); or can prevent Y's satisfaction, or threatening that NotP (negative incentive). But:

<2> *given that X controls positive/negative incentives for Y, she can use these incentives for inducing Y to do or not to do something (Ay). X gets an "influencing-power" on Y.*

However, there are some conditions for that:

- X has to know about her "power-over", otherwise she cannot "exercise" it, use it on purpose for influencing Y;[4]
- Y has to know, to believe about X's "power-over" his goal; and about X's request or expectation about Ay.
- What X wants to impose/induce in Y must be less costly for Y than what she promises or treats to Y. Y's dependence must be more important (value of P, no alternatives, etc.) than the costs of the required action Ay.
- Y must be able and in condition (must have the "power-of") performing the required/induced action Ay (and X and Y have to believe so).
- X is interested in some outcome of Ay; some part of Ay outcomes realizes some goal of X (Gx: that Q).

1.1 More complex relations

Of course, this basic principle is too simple; there are many important aspects in the Dependence relations that introduce complexity in the derived power relations. Let’s just mention some of them.

- The Dependence of Y on X can be more or less strong; this depends on the value of the goal Gy for Y, and on the alternatives (OR Dependence) Y has in that context/network. Given the “degree” of Y’s dependence on X, there is a different degree or strength of X’s “power-over” (and “influencing power”) over Y.
- An OR-dependence link [3] [9], where Y has alternatives reduces X’s “power over” Y, since X has potential competitors; has not the “monopoly” of Y’s need. X gets the power of promising, but not so much the power of threatening Y (see below).
- An AND-dependence link, where Y needs, for his goal Gy, both X’s action/resource and Z’s action/resource, makes the derived power relation really complex: since X’s exercise of her power requires the coordinated use of Z power. X actually depends on Z as for her “power over” Y (they have a "co-power over", see below).

We will put aside here these more sophisticated relations, since we want to focus here on other dynamics and issues.

2 POWER TRANSFER AND APPROPRIATION

Since/if X decides to resort to Y, to use/exploit Y's "power-of", this means that - in some measure - she is

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Our title is a tribute to Ingmar Porn and his book on “The Logic of Power”, although here there is no formal logic at all and “logic” is used in the other theoretical sense.

[2] I’m grateful to Rosaria Conte, Tarek El Sehity, Luca Tummolini for relevant discussions and hints on these issues.

[3] In our vocabulary "incentive/reward power" is just another term for "power over"; since "power-over" is not the power of control (the sociological power) over another agent for inducing him to do or not to do something (this is for us the "influencing or command power"); but it is just the power-over his Goal: frustration or satisfaction.

[4] Awareness is a condition for a full power; without awareness I cannot really "exercise" it for my purposes. [1]

dependent on Y (as for that outcomes of Ay); has not the "power-of" achieving alone that outcome/goal Q. [5] Thus:

> **<3>** *X transforms her "power-of" P [Gy] (for which Y is dependent on X, and X gets "power-over" Y) in her "power-of" Q [Gx], although "mediated" by Y*

X's "power-of" P ==>[6] X's "power-of" Q

X was lacking the "power-of" Q, but thanks to her "power-of" P (relevant for Y) she gets a new power for her own goals (*power expansion).*

X has appropriated a "power-of" of Y for her own goals; she controls it; she has expanded her powers: now it is (in) her power.

Y's "power-of" Q is now appropriated by, transferred to X (especially if X can systematically control it, and induce Y to perform the needed action - See "subjection" definition).

X does not simply acquire one "power-of" Y but a sub-set ***n*** of Y's powers **N** (fig. 1):

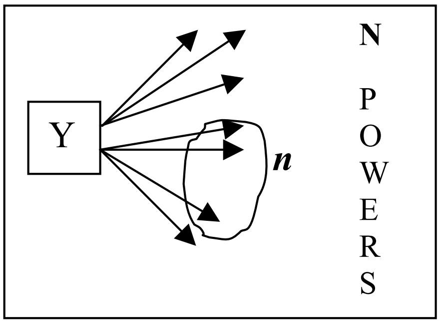

Figure 1.

n = all those powers of Y whose cost (in performing the action) is inferior to the value of the incentive controlled by X.

Thus, X's powers (power-of) increase in such away: + ***n***y.

3 PROPAGATION & ACCUMULATION

Suppose now that X knows that Y's "power-of" (appropriated by X) is a possible reward/incentive for Z; that Z likes or worries about the outcome of Y's Ay, that is: Y's "power-of" is a "power-over" Z.

X may be interested (have the goal) to use/exploit Y's power in this direction; for punishing/rewarding Z, or as incentive for Z. This might be X's goal Gx in relation to Ay outcomes: to use it towards Z.

Since Y's exercise of his own original "power-of" is now under X's control (induction or inhibition) X gets a "power-over" Z.

X's "power-influencing" Y ==> (mediated by Y "power over" Z) ==> X's "power-over" Z ==> X's "power-influencing" Z

> **<4>** *X gets a new "power-over" and "power of influencing" Z, just thank to her "power of influencing" Y*

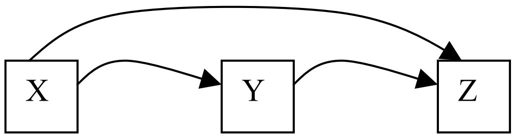

Figure 2.

And so on. X's powers (power-of) increase in such away: + ***n***y + ***n***z ....

Notice 1. We have *derived* X's "power of influencing" Y (etc.) from her "power-of" and "over" some goal of Y (reward, incentive).

However, what really matters for this vicious (or virtuous) circle is the "power of influencing" Y, on whatever basis. The "power-over" is in fact only one possible basis and origin of the "power of influencing". Other bases are - for example - Y's imitation, admiration for X; or X's authority; etc. If X has for whatever reasons an "influencing power" on Y, X can exploit Y's powers and "power over" Z, for acquiring and exercising "power over" and "of influencing" on Z.

Notice 2. Not necessarily Y would be able to personally use his power over Z; X might be able to prevent that: thus Y does not fully has his power, does not really "dispose" of it. This makes Y's powers even more powers of (under the control of) X. (see 4.)

Notice 3. X can use Z powers both *against* Y or *pro* Y. Not only:

Figure 3.

But also

[5] This might be a case of "Weak Dependence": X is able to perform action A by herself, but she decides to rely on Y. In our theory, "weak dependence" is a sub-case of true Dependence. In fact, when X is only "weakly depending on Y" she fully depends on Y for the large/broad goal G' but not for G (sub-part of the outcomes of G'), that she is able and in condition to achieve alone.

[6] This arrow just represents a "generates", "produces" relation.

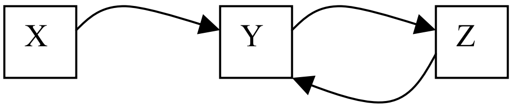

Figure 4.

Let's supposed that Z has – in turn – possible "power-over" Y, in this case X (thanks to Z) strengthens her power-over Y.
X can now use her own, Y's, and Z's powers *pro* or *against* W. And so on.

4 NOT AGGRESSION ONLY

A power is not *per sé* good or bad, benefic or malefic. Any power has two faces: a benevolent and a malevolent one. X could either let/make Y realize his goal, or frustrate it. However, since the goals of Y can be achievement or aversion goals there actually is an important asymmetry in Powers and Power relations.
In any kind of *Power-over,* X can frustrate or satisfy Y; can make him happy or unhappy. However, in one case "happy" means some gain, while unhappy means no gain, while in the other case "happy" simply means no harm, and unhappy means a loss.
Thus, we have to distinguish between:

Harm/Loss Power-over (HP):
the Power of X of producing a loss, some harm to Y.
Where the "good" is do not lose, do not receive harm; to remain as before. HP gives a choice (two possible actions/outcomes): harm /no-harm to Y.

Gain Power-over (GP):
the Power of X of producing a gain for Y.
Where the "good" is increasing Y's wealth, welfare, and the "bad" is to remain as before. GP gives a choice (two possible actions/outcomes): gain /no-gain to Y.

Given the psychological asymmetry of subjective value between losses and gains, and given that possible harms elicit an "avoidance" response, that has priority, the HP is more influencing, persuasive. And the situation is perceived by Y as coercive, not really a "free" choice.
X is perceived by Y as threatening, hostile, bad. In the GP X is perceived as more cooperative, promising.[7] However, the HP activates a tendency to escape from that relation, and searching for promising, improving relations.

[7] Actually this is the distinction between "true/deep" Promises vs. "deep" Threats. [5]

Frequently both forms of power coexist and are exploited for building stable relations.

4.1 Active/Passive Power Exercise

Another fundamental asymmetry is between power exercised by performing an action (*Active Power*) vs. power exercised by abstaining from an action (*Passive Power*). For example, in HP X can actively produce Y's harm (for example, by beating him) or can simply let happen something bad for Y that she might prevent (for example, do not providing medical care to Y).
While crossing the two distinctions HP (with its two moves/results) and GP (with its two moves/results), and Active/Performing vs. Passive/Abstaining we find 4 x 2 interesting power-relation forms: See Figure 5.

	ACTION A producing HARMS	**ACTIVE** (by doing A)	**"aggression"** ***HARM****: Y worst than before*
HARM-POWER	**idem**	**PASSIVE** (abstaining from A)	**"sparing"** ***NO HARM****: Y like before*
	ACTION A' preventing HARMS	**ACTIVE** (by doing A')	**"protect"** ***NO HARM****: Y like before*
	idem	**PASSIVE** (abstaining from A')	**"do-not-protect"** ***HARM****: Y worst than before*
	ACTION A producing GAINS	**ACTIVE** (by doing A)	**"good-to"** ***GAIN****: Y better than before*
GAIN-POWER	**idem**	**PASSIVE** (abstaining from A)	**"no-good-to"** ***NO GAIN****: Y like before*
	ACTION A' preventing GAINS	**ACTIVE** (by doing A')	**"no-good-to"** ***NO GAIN****: Y like before*
	idem	**PASSIVE** (abstaining from A')	**"good-to"** ***GAIN****: Y better than before*

Figure 5.

5 THREATS VS PROMISES ASYMMETRY

To get influence X has to *signal* that she disposes of the "power-over" Y (Z, W, ...). But not necessarily she have to perform the relative (threatened/promised) action:

A **Threat** will be successful only if the "power-over" will NOT be exercised. In case of obedience the threatened action will NOT be performed (threat kept) [2].

A **Promise** works if X's "power-over" Y is believed, although, in case of obedience, the action must be also actually performed, spent (promise kept).

This is the serious economic *asymmetry* between threat and promise. Threat is very convenient for the influencer (threatener); if it works she has to spend nothing! However (also for this reason and because - as we said - it focuses on losses and harms) the threat induces more rebellion and escaping, and thus requires heavy costs of surveillance, anti-rebellion, etc.

6 "RIGHTS": THE POWER OF THE WEAK

There is another fundamental and basic form of power-acquisition from the others, of the "empowerment" of X by the others; not just when the others pass their powers to X, give her the control over their personal powers (skills, competences, resources); as we have just seen.
Does X (with her personal skills and resources) really/fully have the "power-of" G1 when she is in a social context, that is, in a "common word" with other agents, with the possibility not only of "positive interferences" (realizing goals by exploiting the others action and outcomes) but also of "negative interferences" [3]: when the others' activity can create obstacles to X's actions and block the achievement of X's goals?
In this context, paradoxically X's basic "power-of" G1 depends on the others and depends on X power to prevent or block Y's interference, in order to freely exercise her abilities and accessing her resources.
X's "influencing-power" over the others gives X the full "power-of" G1. The "influencing power" is a basis of the "power-of"; the individual "power-of" acquires a social connotation and ground.[8]
In this perspective, there is another crucial mechanism not based on X's deterrent power and direct influencing the others. Suppose that X would not be able to block Y/Z or to prevent Y/Z's interference; however, if Y and Z spontaneously refrain from interfering on X's exercise of his "power-of" G1, and simply "let", "permit" to X to achieve her goal, they are actually "empowering" X in a peculiar way. Not by transferring to X their own "powers-of" but practically "permitting" to X to do a given action, to access a given resource.
A tacit agreement on this, and shared expectations about, are the basis of "rights" and their "recognition" not at the legal/formal level but at the interpersonal level, when "rights" are just precedence rules claimed by X and acknowledged (and given) by the others. "Rights" are a fundamental form of social "granting/acquisition" of "power-of*": the power of the weak*, of those who wouldn't have coercion power over the others. And this remains true also for the legal granting of rights, where the "authority" takes care of the weakest. Not only the strongest ones can acquire powers from the others, by violence or alliance, but also the weakest.

7 DEFINITIONS

X's impressive power of control and "power-of" are due to the <u>subordination</u> (see below) of the others, to the <u>alienation</u> (see below) and <u>concentration</u> (§ 3, 4, 9) of their powers.

7.1 "Subjection"

Y is "subject", subservient, obedient to X, he "submits" to X, when he decides of not opposing, resisting to or negotiating case by case X's influencing acts (that become "orders", "commands"), but of just executing them, just obeying.
Since he has *accepted* (taken note or desired) that X's has a "power over" him (rewards/incentives; either threats and harms, or prizes) and can influence him.
Y does not simply have time by time the goal:
(i) If X asks Ay ==> Goaly Ay

but he has the generalized goal:
(ii) For any value of Ay (If X asks Ay ==> Goaly Ay)
and (i) becomes just an instantiation of (ii).

In other terms, Y passes from a local, occasional *Commitment* towards X [4], to a generic one about obeying to X or at least cooperating with X/Group.
And X passes from an occasional promise or threat towards Y (for influencing him "now and here") to a general *Commitment* to protect Y (or do not harm Y), to reward Y, or to share with Y the benefits of the collective power. They rely on reciprocal fidelity and loyalty.

7.2 "Alienation"

Is Y aware of the fact that X has (disposes of) his powers? That X has powers because Y passes to her his powers, gives his power (by his "submission")? In a sense, is Y aware of his "loosing" that power. Y does not fully understand that X is *dependent* on him, not only the other way around [9]. Y just realizes, considers that he does

[8] This is for us the meaning of Hobbes' claim that: *"power is simply no more but the excess of the power of one above that of another"* [10] (I.viii.4), and the reason of Weber's notion of power immediately defined in a social perspective and over the others.

[9] Also because this dependence is asymmetric, and because in our common sense the concept of "dependence" has a hierarchical connotation. Of course, this is a partial analysis of "alienation".

something *for* X in exchange for something from X (avoiding harm, getting a prize). He doesn't fully realize the "transfer" of power mediated by X's "power-over" Y.
Especially with a group G Y operates in/for G in order to give power to G (or X, the boss) and to receive some benefit or valuable identity. He doesn't really perceive that without his role and contribution X/G would be nothing. Also because there is a cognitively difficult "collective" phenomenon, and it wouldn't be really true that, without Y, X or G would be nothing. Only a collective defection would make collapse X/G and would reveal its derived and indirect power. He perceives G and X as *per sé* endowed by the power that he (the members) is/are actually conferring/passing to it/him.
The individual defection of Y would in fact just expose Y either to losing gains or to retaliation; psychologically confirming that the power is of the G/X, not "mine".

8 CO-POWERS AND THE MULTIPLICATION OF POWERS

The impressive accumulation and multiplication of X's powers are not only due to the indirect/mediated use of the personal powers (§ 2, 3 & 7), but to another phenomenon: the existence of "co-powers".

> **<5>** *Not only X acquires part of Y's powers (**n**y) and of Z's powers (**n**z), ... but she acquires powers that neither Y nor Z individually have; powers they have only if/when they act together and in a coordinated way.*

For example, individually Y or Z or W have no power (or very limited power) of intimidating/threatening J, but together (as a gang) they get this result; analogously, the working capacity of Y+Z+W is greater than the sum of Y's individual isolated work and Z's individual work and W's individual work.
To be true, even the fact that Y has an "influencing power" on Z and can exercise it, becomes a "co-power" after that X has appropriated it and controls it. In fact, Y's depends on X as for performing or not the action, exercising or not his "power over" Z. Thus this power and the consequent "influencing-power" being transferred to X and mediated by Y is now a "co-power" of them.
However,

> **<6>** *An effective "co-power" presupposes coordination of the actors' actions and powers.*

This clearly is one of the possible benefits and functions of the power concentration under X and of "command" positions.
In order to really have a power and exercise it, as we said, X has to *know* that she disposes of it; the same for coordinating different powers. The same for the others: they have to know/believe to be dependent on X and accept to be subject to X.
However, not necessarily the coordinated co-operators know of each other (this depends on the need or not for a decentralized coordination among the executors). And this is an additional reason for the non-awareness of the co-power (*alienation*). In any case, as we said, X has to *signal* that she disposes of the "power-over" Y, Z, W, ...

9 POWER LIKE MONEY

As we said, X's "Power-of" for Gy (P), and the *following* "Power-over" Y and the acquisition of Y's "Power-of" Gx (Q), is in fact a *Transformation* of the power. The original power of X was good for realizing P, but now – thanks to the social mediation, to its utility for and acceptance by other agents (with different goods/skills/resources) – it transcends its original "use" and "use value":

> **<7>** *The original power becomes valuable, usable for new goals, that is, for new "uses". It becomes multi-purpose or better for an* ***open-use****: it might be useful for an open set of potential uses/goals depending on the powers of the other accessible agents.*

Similarly to money (which actually is a very special form of *Power-over* the others' goals and thus of *Influencing-Power*) *Power-over* is now *a "means" for whatever "end"*. And it can also be "stored", not immediately exercised and spent, in view of future possible uses. It also acquires an "exchange value": X can exchange her *Power-of* or *over* in change on ***n*** possible Powers of the other agents. Her power is on a market of Powers circulation.
This is the main reason why social power becomes an end *per sé*, a final motive (not immediately instrumental, not already "in view" of some use), and why there is a lot of competition for acquiring "power-over" and "of influencing". X and Z might compete with each other for acquiring Y's powers and Y's subjection, like for any other scarce resource, especially if multi-purpose.
The controlled powers of the others are a fundamental "capital" to be cumulated, disputed, invested, and exploited for social (or socially mediated) goals.

10 APPENDIX: "TRIBUTE" TO AXELROD

This power concentration process is very close (and in part captured by) Axelrod's "Tribute model" about the aggregation of political actors [6] [7]. A part that Axelrod interest is more restricted and specific: modeling the emergence of political entities and of nations; it is true that his model captures some of the main issues of power aggregation (but it doesn't explicitly focus very much on

power transfer, dependence, etc., while dependence relations apply also to macro-agents and institutions [8]).
Axelrod well defines the emergence of a new level "actor" "entity", agent, in terms of:

(a) subordination and control of the other actors, rather stable, without rebellion, etc.;
(b) collective action, cohesive attitude towards the outside actors; protection of the weak members;
(c) recognition of the new emergent entity as an unitary agent, by the others.

Moreover Axelrod stresses the "commitment" between the parties and its acquisition and loss.
The main difference is that he lacks a basic general theory of power forms and dynamics. On the one side, he presents a rather unilateral, unbalanced view of "power over" the others (and thus of the "concentrated" power). The emphasis is only (mainly) on threat power, on violence, on harms and losses or on protection from that or renounce to that. Much less importance has the aggregation (and even subordination) due to buying favors and services, to corruption, to remuneration. When Y depends on X (and X gets "power-over" Y) this is not primarily related to harms, aggression, protection, ..; it is equally related to Y's desires, needs, ambitions satisfaction and to possible "prizes". On the other side, the role of immaterial good looks underestimated: identity, recognition, membership, role, ... power increment, power-over, hierarchy, security, ... as valuable and rewarding *per sé*. Not just "wealth", or aggression power (military force).
We need a theory of power dynamics based on a adequate theory (spectrum) both of human motivations and of the various sides and forms of power.

11 CONCLUSIONS & FUTURE WORK

This paper explains how the power of an agent creates social power over the other agents; how an agent acquires new powers, and a given power becomes a different power; how a power is transferred from one agent to another one and accumulated; how co-powers require coordination. What is power 'alienation' and 'subjection'.
Not only new powers are acquired by an agent but those powers are multi-purpose: a fundamental "capital" to be cumulated, disputed, invested, and exploited for social (or socially mediated) goals.
It is also argued that power of influencing and subjection are not only due to harm-power and threats; that there is a fundamental economic asymmetry between promising and threatening, and that there are several (active and passive) ways for harming or benefiting somebody.
This preliminary – somewhat systematic but informal - theory about power transformations, transferring, accumulation, transitivity, multi-purposiveness, etc. obviously should:

- on the one side, be formalized in some formal logic of action;
- on the other side, be object of social simulation studies and network dynamics modelling.

Everybody focuses on "cooperation" or on "games", but the issue of dependence and power relations and dynamics is the real background and preliminary theory for that [9].

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How detailed should social networks be for labor market's models ?

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Abstract. Many empirical studies emphasis the role of social networks in job search. The social network implied in this process is known to be characterized by complex properties, including communities, homophily or more or less strong ties. Nevertheless, previous models of the labor markets fail to capture the complexity of social networks, as each specific network requires the development of specific algorithms. In this paper, we rather rely on an independent generic network generator for creating detailed networks describing friendships, colleagues, communities and various degrees of connectivity. We build a simple model of the labor market in which individuals find positions solely through their acquaintances, and update their network when being hired. This original experimental setting facilitates the analysis of various characteristics of networks on the labor market, including various size, more or less friendships, or the impact of communities. Experiments confirm the "strength of weak ties" phenomenon. However, the initial characteristics of the network like communities are shown to be destroyed by the implausible mechanisms described into this simplistic model; this suggests that the impact of plausible networks on models' dynamics may only be studied when the mechanisms of this model are plausible as well - in other words, "a model is only as descriptive as its most implausible components".

1 Introduction

1.1 Empirical evidence on the use of social relationships in labor markets

Field studies on job search highlighted several stylized facts. First, (Stylized fact 1), it appears that searching and finding a job implies the use of social acquaintances to retrieve information [16, 17] (see [10] for a review). As an illustration, Granovetter's studies indicate that about 50 percents of jobs are found through friends, relatives and other social contacts [7]. As done since decades in sociology [23] and economics [11], this social structure is commonly represented using the social network metaphor: each individual is assimilated to a node, with communication links being represented as edges in this network. From an economic viewpoint, the communication of job opportunities through social relationships may lead to incomplete information in the market, thus possibly leading to a market efficiency lower than the optimal one.

Secondly (stylized fact 2), all ties are not used in the same way by job seekers, nor lead to the same information. Since the famous Granovetter's studies on job search [9, 8], it became common to distinguish weak and strong ties; strong ties in a social network reflect frequent interactions between individuals, while weak ties typically lead to less frequent and less personal relationships. Moreover, strong ties are more local, because they are mainly created and maintained because of common workplaces (co-workers), life-places or other activities (near family, friends); typically, the clusters observed in social networks are mainly made of strong ties. Weak ties are more random in the network; they correspond to old friendships born at school or far family. Granovetter observed that despite of long distance and rare interactions, weak ties are more efficient for finding job opportunities than strong ties: strong ties correspond to less diversified people who may communicate easily but receive the very same information, while weak ties link more different people exposed to different types of information. These observations were replicated on different countries and populations (see [19, p.5] and [10] for detailed reviews).

The third stylized fact (stylized fact 3), shared by all empirical studies on social networks [6, 23], underlines the complex nature of these social networks. First, the position of agents on their social environment is far from being random; at the dyadic level, it appears that people tend to bond together when they have close socio-demographic characteristics or interests (homophily), or more generally that the existence of a social tie depends on the properties of individuals (assortativness). The use of social acquaintances to search for jobs often changes with location and demographic characteristics. Living in the same location increase the probability of co-working, as do similar socio-demographic characteristics [2]. Moreover, complex patterns are robustly observed in real networks at the scale of the triad (strong clustering or transitivity rate, intuitively corresponding to the "friends of my friends are also my friends" effect). The recent stream of statistical analysis of large networks [15] also highlighted network-scale properties of real networks, including the frequent presence of biased distribution of degree of connectivity (most people have few ties, while few trust a big number of relationships).

1.2 Previous models of the labor market with information transmission

Models of the labor market progressively took into account the stylized facts described before. First, the use of social networks for conveying information was added to the models[3]; Montgomery [14] highlighted how heterogeneity in the efficiency of job search could arise from structural characteristics. Cahuc and Fontaine [5] showed that these networks lower the efficiency of the market and lead to the existence of several local equilibria instead of a global one.

Some authors described different kinds of links in their networks (multiplex networks) in order to recreate weak and strong ties. No-

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tably, Tassier [20] developed an algorithm that enables to tune the proportion of local vs. random social links across the population. This study enabled the reproduction of the Granovetter's strength of weak ties effect, and proved the importance of the ratio local/random links on market efficiency.

Some recent models also described attributes of agents, that are created according to their position in the network, in order to comply with stylized fact 3. Bramoullé and Saint-Paul [4] describe homophily on salarial status by giving a higher probability for two individuals having the same employment status to be linked together. Tassier [19] used an ethnicity attribute for the agents, and studied the sensitivity of the market to more or less overlapping between communities (see also [10] for a detailed review of existing models). Unsurprisingly, the impact of the social network appears to be strong at different scales: the initial position of agents over the network impacts their probability of finding a job, communities may be more or less efficient depending to their endogenous structure, while all these local phenomena also create different levels of efficiency at the population scale.

In short, both empirical and theoretical studies agree on the impact of local properties of networks on labor markets' efficiency. Models taking into account the complexity of networks remain rare and limited: they only assess the impact of one specific detail (ethnicity, spatialization, etc.) on the market's dynamics. Many other properties of real networks, like the frequent presence of biased distribution of connectivity, remains unexplored. The difficulty to generate "plausible" detailed networks probably constitutes an explanation of these choices.

1.3 Open questions

Despite of the increasing number of models devoted to the study of the labor market, several questions remain open, including:

- *How to generate "rich" networks*, that is networks having several of the properties observed in real networks ? This generation is mandatory for the study of the impact of these properties to the dynamics of the labor market. However, generating such a network constitutes a difficult question in itself, which was not solved for the previous models, thus limiting these studies on networks' impact to only one attribute or three links types. In short, this technical limitation on networks generation forbids the computational study of the impact of these numerous complex properties.
- *How realistic should an initial network be* for models of the labor market in which the network evolves ? Previous works studied either the impact of a static and detailed network on simulations (e.g. [19]), or the evolution of a simple network ([18]). Coupling these dynamics may increase (or lower) the sensitivity of the models to the initial network. Once the sensitivity of such a model is known, the incorporation of social networks into a descriptive model of labor market may become possible.

1.4 Outline

In the next section we will describe the two components of our experimental setting: the use of a generic network generator for constructing the initial networks and a simple model of the labor market. In section 3 we show some results of experiments focused on the efficiency of various link types for finding a job, and on the evolution of networks. As discussed later (4), these experiments suggest that using "rich" initial networks is useless if the dynamics of the model is unrealistic enough for changing its initial structure.

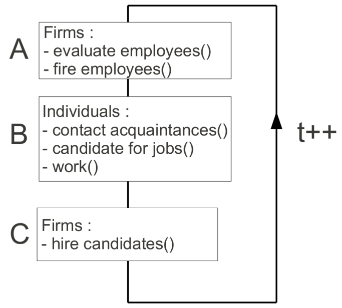

Figure 1. A whole cycle in the simulation

2 Model and Experimental setting

2.1 Model of the labor market

We keep our model as simple as possible in order to catch the most fundamental aspects of the labor market itself. We present here the agents participating in the simulation, its protocol and properties (hypothesis) that we use.

2.1.1 Agents

Two types of agents are used in the model: Individual agents and Firm agents. Firm agents propose jobs and hope to fill them with Individual agents who propose their labor and hope to occupy the jobs.

An **Individual** agent can be in one of these two states: An *Employed* agent is currently occupying a job. *Unemployed* agents do not have a job, but they are looking for one. An Individual agent is described by its gender, state and the acquaintances it has.

A **Firm** agent offers jobs, hires and fires Individual agents. Jobs are represented as objects belonging to Firms. A job can be in either *Filled* (An Individual agent is currently occupying this job) or *Vacant* (the job is not filled and the Firm agent would like to hire an Individual agent to occupy it). A firm agent is described by its size which is the number of jobs (vacant or filled) it possesses.

2.1.2 Protocol

A cycle in the simulation takes place in 3 parts (see Figure 1). First (A), Firms lay off some of their employees with a random probability *p_fire (=10%)*.

In the second part (B) Individual agents interact. If they unemployed, they look for a job. In this basic model, individuals may only find jobs using their social acquaintances (as done previously [1]). The Individual agents may contact their friends, colleagues etc. Then they candidate to all the vacancies they encounter.

In the last part of the cycle (C) Firms iterate all their vacancies. If a vacancy got no candidature requests, it stays vacant. When a vacancy has several candidates, the Firm chooses randomly a candidate to be hired under the condition that it will not hire an individual who it just laid off. As soon as an individual is hired, his new colleagues are added to his set of "colleague" acquaintances; however, in order to forbid agents to know the entire population, colleagues are then removed randomly from this list of acquaintances, in order to keep the list of colleagues to at most $max\_colleagues$ (parameter). As a consequence, individuals remember past colleagues from past positions; however, the older the colleague, the higher the probability to break this tie.

2.1.3 Hypothesis

The structure of the simulated labor market and the interactions between the agents follow the hypothesis listed in table 1.

1. The numbers of jobs and Individual agents in the simulated labor market are equal. That means the a situation of zero unemployment is possible.
2. The numbers of jobs and Individual agents are constant throughout the simulation. That means that Firm agents cannot destroy nor create new jobs and the Individual agents do not age nor enter or leave the simulation.
3. Firms are passive in the process of job searching. They do not advertise their vacancies and therefor do not spend time nor money in trying to improve their chances of filling their vacancies.
4. Individual agents in the state of unemployment are constrained to look for a job. They may not stay in unemployment without an active action of job-search (leave to inactivity).
5. Unemployed individuals contact there social networks: Spouse, old colleagues, friends and spouse's colleagues. In order to look for vacancies. All vacancies encountered are listed and a candidature is sent to each one of them.
6. Unemployed individuals candidate to all vacancies they encounter.
7. In order that an agent be able to communicate vacancies to a job-seeker, he has to be employed. In this case he communicates vacancies available in the firm in which he is employed.
8. Job selection (matching) is purely random; this hypothesis simplifies the analysis of the dynamics, as all workers have theoretically equal probabilities to be hired, the only bias being induced by their position on the network (as done in [19]).
9. Worker agents are fired in a pure random way. They stay employed until they are fired. They may not quit the firm.

Table 1. Hypothesis

2.2 Social network

The YANG network generator [21] stands as a generic tool dedicated to the generation of plausible networks for social simulation. Its principle is to accept rich parameters in order to reconstruct plausible networks from rules at the local scale. Resulting networks are multiplex (different kinds of relationships), mixed (directed or undirected links) and attributed (each individual has attributes and is positioned in a plausible social neighborhood). We set the parameters in order to (i) generate a population of agents, which includes both individual and firms, and (ii) to create an initial matching between firms and agents, as well as the numerous social links. Note that all individuals are assumed to be potential workers.

Link types are provided as couples {name, directionality}. In this application, we use here as link types: {{married, undirected},{worksInFirm, directed},{colleagues, undirected},{friends, undirected}}.

The network generator accepts as many *discrete agents' attributes* as desired. We define here the attributes listed in Table 2. Attributes of agents are described in YANG as random variables in a Bayesian network. This formalism enables the description of interdependencies between attributes. Probabilities associated with these variables are defined as follows: agentTypes takes value 'firm' with probability 0.1 and 'individual' with probability 0.9, leading the generator to create one firm per nine individuals. In the same way, gender take

attribute	domain	depends on
agentType	{firm, individual}	{∅}
gender	{notRelevant, male, female}	{agentType}
salarialStatus	{notRelevant, employed, unemployed}	{agentType}
auto_friends_degree	{0..10}	{agentType}
auto_wedding_degree	{0,1}	{agentType}
auto_eco_indegree	{0..20}	{agentType}
auto_eco_outdegree	{0..20}	{salarialStatus}

Table 2. Agents' attributes. Attributes in italic correspond to the degree of connectivity for generation rules.

'male' and 'female' values with probability 0.5 for individuals and value 'notRelevant' for firms. At initialization, 10% of the workers are not tied to firms and will have to find a job[4]. Attributes in italics in Table 2 correspond to the degree of connectivity for various generation rules described below. In practice, the degree for friendship (attribute *auto_friends_degree*) will be set to 5 or 2, depending to the experiments. In and out degree of connectivity for the matching of firms (*auto_eco_indegree* and *auto_eco_outdegree*) respectively describe the number of links getting out of an individual (1 if employed, 0 else) and going in a firm (9 for all firms in the first experiments).

rule name	method	principle
wedding	attributes	create links 'spouses' between males and females for 80% of agents with max degree 1
match	attributes	create links 'worksInFirm' between individuals having 'employed' as salarialStatus and firms
colleagues	transitivity	when an agent A1 'worksInFirm' A2, and another agent A3 'worksInFirm' A2, then create a link 'colleague' A1 and A3
friendsRandom	attributes	create links 'friendship' between individuals in pure random way

Table 3. Generation rules

The last parameters of the generator are the *generation rules*[5], which describe how the links are actually created in the population. YANG accepts two types of generation rules: "attributes rules" refer to generation rules that match two agents depending to their attributes, while "transitivity rules" propose the creation of links at the triadic scale by transitivity. We define the generation rules described in Table 3. The spirit of these rules, which will be applied in this order, is to create wedding links; then, to attribute to each worker a firm; then, to create links between all the colleagues; last, to create friendship links randomly across the population.

The YANG network generator uses all of these parameters for generating random networks of size N. It first creates the whole population, each agent being given a combination of the possible attributes values. This population is stored in an SQL database. Then, the generator applies all the generation rules, by retrieving agents that may be tied together by SQL set operations on the population. The software that implements the generator also provides dynamic visual-

[4] Which will generate initially 10% of unemployment.
[5] Note that attributes rules always implicitly take into account the degree described before as an attribute of the agent. Laso Note that some of these rules are changed in some experiments.

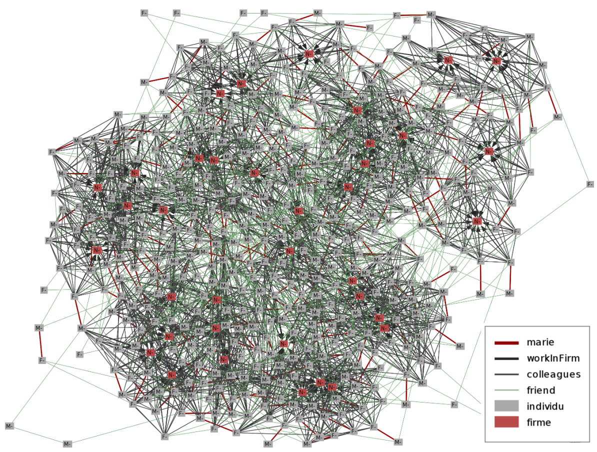

Figure 2. Example of a initial network used as a parameter.

ization of the network generation in order to check their plausibility. More details on this algorithm are provided in [22]. The detailed parameters are provided as supplementary material for reproduction purpose[6]. An example of a resulting network is depicted in Figure 2.

It is important to note that, as this network generator is random, the generated population may be slightly biased; for instance, the actual proportions of agents and firms may be 85/15 instead of the theoretical 90/10. As a consequence, the number of positions and individuals in sometimes not strictly equal in generated networks. To solve this problem, when networks are loaded, open positions are removed randomly if positions are too numerous, or open positions are added if workers are too numerous.

2.3 Implementation

Our model was implemented in Java (1.6) under the platform Repast[7]. In order to get the results we present here we used 1 000 agents : 900 Individuals and 100 Firms. The generation of the simulations took place in 2 stages. First we generated the network which defined the number of agents, their characteristics and the relationships between them. Then we used this network in order to initialize the population of agents that interacts during the execution of the simulation. During this execution we gathered several statistic data which we will present and analyze in the next section.

3 Experiments

At initialization, the state of the population depends on the construction of the model: $\sim 10\%$ of individuals are employed by a firm; the rate of open positions is exactly the same. At each step, each agent has a probability of 10% to be fired (firing rate fixed to 0.1). Agents initialized as unemployed attempt to find one of the available positions in their social neighborhood.

Once unemployed, each agent candidates through his 5 friends (or 2, depending to the experiment), his spouse (if married) and his 5 colleagues (with $max\_colleagues$ set to 5). In practice, the colleagues of the position he last quit are useless, as this firm cannot hire him immediately. Also, candidatures are only allowed at degree 2 (individuals candidate to positions available in the firms of their neighbors). If a position is open in its neighborhood, the individual may be hired by this firm; in such a case, he discovers several new colleagues (and forgets few old ones, such that his total number of colleagues remains under $max\_colleagues$).

As an individual always keeps his initial friends, and remembers some old colleagues, he accumulates a set of acquaintances which is more and more efficient to find positions in new firms. Experiments prove that with this dynamics, he may even “travel” across the networks while he discovers new open positions, new colleagues, and so on.

During the experiments, we measure the unemployment rate, the average number of firms visited by individuals, and the efficiency of each link type for finding a position.

3.1 The incontestable strength of weak ties

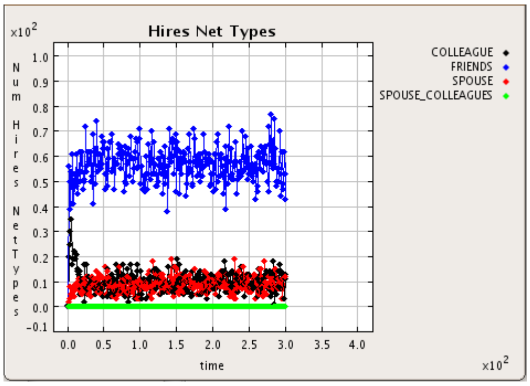

Figure 3. Efficiency of the various link types during a typical run of the model for 5 friends and 5 colleagues.

parameters	Unempl-rate	links efficiency colleagues	friends	spouse	firms count
same size for firms					
5 friends	1.9%	36.9%	54.7%	8.5%	16.57
2 friends	3.0%	55.3%	32.3%	12.3%	13.98
fat-tailed distribution of firms					
5 friends	2.1%	38.1%	53.0%	8.9%	16.62
2 friends	3.2%	54.6%	32.9%	12.4%	13.91

Table 4. Unemployment rate, efficiency of the various link types, and average number of previous positions per agent, for various combinations of parameters.

In this first set of experiments, we explore which links, in this simple model of labor market, enable people to find positions after being fired. The first simulations are run with as many friends as colleagues (5). We observe for each simulation the unemployment rate, the efficiency of each linktype for finding a position, and the total number of firms each individual worked for. As depicted in Figure 3, a typical run starts with a stabilization phase in which agents which were initialized as unemployed in the network search and find jobs. Then, the unemployment rate stabilizes around a certain level (which reflects the market’s efficiency) whilst the agents are being fired and

[6] Note for reviewers: sourcecode and parameters will be soon shared on a website like openabm for enabling reproduction.

[7] http://repast.sourceforge.net

search for positions in other firms through their social acquaintances. In order to compare several parameters, we measure the aggregated value for all the indicators after 300 steps for one hundred simulations for each set of parameters (one different network is loaded for each simulation).

Simulations reflect the Granovetter's strength of weak ties: even if individuals have the very same number of friends and colleagues, they actually find most of their positions (55%) through friendship, which is twice as much through colleague links (35%). This effect is explained by the position of these neighbors; while colleagues are mainly aware of the positions available in the former firm in which the individual worked (which can no more hire him), friends are dispersed randomly across the population. Even when we decrease the number of friends to 2 (with 5 colleagues), these links still perform relatively better than colleagues links (Table 4). In this last case however, some unemployed individuals fail to find positions, leading to a higher unemployment rate of 3.0%.

All the firms in these experiments had exactly 9 positions; we now experiment a fat-tailed distribution of firms[8] in order to assess its impact on the market. As reflected by aggregated results (Table 4), the unemployment rate increases slightly: open positions are transmitted by the workers of the firm, which are more numerous for bigger firms. Except this minor change in unemployment rate, the dynamics of the model does not seem to change by this distribution of firms' sizes; notably, the efficiency of each link type remains similar.

3.2 Describing communities: from order to randomness

Evidence from sociological studies demonstrate the strong clustering of populations (see 1.1). As our experimental framework enables to tune the structure of networks easily, we drive several experiments based on networks structured in communities of different sizes. These communities are characterized by a large majority of social links which are endogenous in each community, with only few links creating "shortcuts" between communities. We expect these communities to lower the ability of individuals to find positions opened in other communities, which would lead to a higher unemployment rate and - as a side effect - a lower number of previous workplaces per agent. Inspired by the work of [19], we first created three main areas with only a few links between them. We expected a higher unemployment rate, which was surprisingly not observed in the experiments. As a consequence, we designed a highly clustered network in order to study this phenomenon.

In this experiment, we add a "community" attribute to agents (both firms and individuals). This attribute takes values between 1 and 100 with equal probability (each community has the same size). Endogeneity is strong in this network: friendships only occur in the same community, as do spouse links. Positions are initially filled by individuals belonging to the same community or to close neighbors. For instance, firms in position 50 only hire individuals from communities 49, 50 and 51. As a consequence, initial colleague links are only created at degree three in these communities. The resulting network constitutes a kind of one-dimensional lattice of diameter 24, as depicted in Figure 4. This network is obviously unrealistic and is only used for understanding why communities appear to have such a low impact on the models' dynamics.

In such a highly clustered and large diameter network, we would expect a lower unemployment rate; even if the lowest possible unemployment rate is reached, it should take numerous steps before agents "move" from one part of this large network to another. One more time, experiments contradict this intuition; the unemployment rate is close to the ones obtained in previous experiments. Individuals also explore the same number of firms (∼14.4 in average), despite of the absence of shortcuts in the initial network.

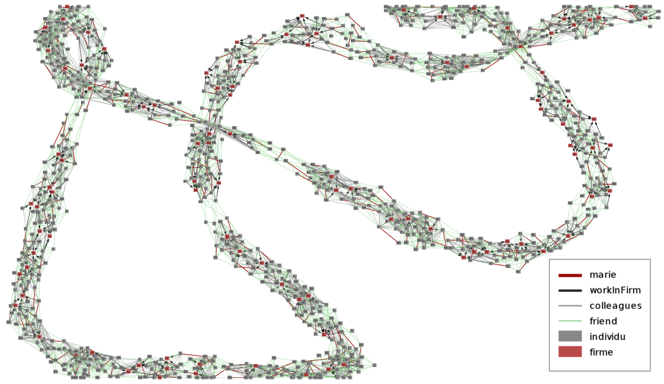

Figure 4. Example of network with one hundred communities.

3.2.1 *Unrealistic dynamics passes over realistic initial networks*

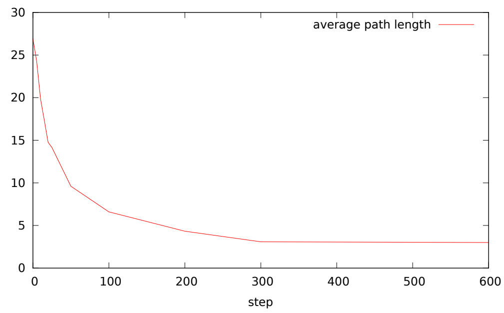

Figure 5. Evolution of network size during steps

An analysis of the social network at different steps reveals how quickly it shrinks (Figure 5). After 300 steps, the average path length in this network is as low as 3.18. Agents changing positions appear to create shortcuts quite rapidly. This leading to a small-world effect already measured in previous experiments [18]. This (possibly) unrealistic drop in network diameter is probably explained by several unrealistic processes in the model:

- There is *no cost for changing community* for agents as would be expected in reality (time, psychological cost, relocation cost) nor costs for repeated change of community (or area).
- Only colleague links are partly changed when individuals are hired; friendship links remain stable and conserve the very same communication power (same efficiency for finding information). In real settings, a higher distance would change these friendship

[8] 50% of the firms have 5 positions, 30% 10 positions, 10% 15 positions, 5% 20 positions and 5% 30 positions.

links, which were first created in the same community, into weaker ties.

This observation may be thought to be an interesting analogy of job markets in which workers are highly mobile and positions are mainly discovered through social acquaintances. Typically, this could be the case of research positions, with post-docs moving from laboratory to laboratory, thus improving not only their own efficiency of finding positions, but also enabling their friends to discover positions in their old laboratories.

Beyond the case of labor markets, this phenomenon underlines an interesting methodological point for agent-based simulation: *the plausibility of a model is as strong as the weaker plausibility of its components*. Providing networks with many details - even if plausible or real - is useless if the dynamics that change this network later is unrealistic. In our example, we increased the plausibility of the initial network, but we did not describe a plausible hiring process; we should haven taken into account both the relocation cost (changing community) and the lower strength of old friendship links.

4 Discussion

In this paper, we set up a framework for exploring the impact of a detailed networks on the dynamics of a simple labor market model. We used the YANG standalone network generator for generating networks having diversified properties such as various link types, degrees of connectivity, firm sizes or the presence of communities. This initial network evolves as individuals discover colleagues when they are hired in new firms. In this simplistic model, interpersonal communication is the only way to discover job positions.

We first studied the *efficiency of the various link-types* used in the model. These preliminary studies confirm the "strength of weak ties" famous phenomenon: as they are created in a random way across the population, friendship links enable individuals to discover open positions in others firms, while colleague links remain focused on the last firms visited by the individual. We have to moderate this observation by the fact that these links were not exactly described as "weak": weak links suppose a small probability to interact, and including such a lower probability would probably reduce their efficiency. Nevertheless, given these observations, using a network generator that enables the description of different link types, appears to be mandatory to build plausible models of labor markets.

We used the versatility of our experimental setting for creating *clustered networks* in which agents' attributes determined their belonging to groups. Surprisingly, the creation of networks, segregated into weakly interconnected communities, did not lead to strong shift in unemployment rate. Indeed, the parameters led to frequent changes in the network (firing rate at 10%). Moreover, the evolution of the network, instead of the initial construction of the network, did not take into account the attributes of agents; as a consequence, this dynamics quickly "changed" the initial structure, making the initial characteristics of the network secondary regarding this evolution. This last observation may be generalized beyond the limited scope of labor markets: in this kind of model in which the network is dynamically changing during the simulation, *the use of a "more descriptive" network is useless if this evolution is based on implausible behaviors.*

Given this first analysis of the evolution of a "rich" initial network into an agent-based model of the labor market, we plan to limit the destruction of the initial network by associating probabilities of interaction to link types. Once the deformation of the network will be limited, further inquires will be driven on the impact of various initial structures to the models' dynamics.

Then we would like to couple these social networks with more descriptive models of labor markets (like [12, 13]) in order to study their impact on the labor market's outcomes studied.

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Considering baseline homophily when generating spatial social networks

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Abstract. Social networks have become an important part of agent-based models, and their structure may have remarkable impact on simulation results.

We propose a simple but powerful approach for spatial agent based models which explicitly takes into account restrictions and opportunities imposed by effects of baseline homophily, i.e. the tendency to build up relationships with others that are similar. The resulting network thus reflects social settings and furthermore allows the modeller to influence network properties by adjusting agent type specific parameters. Especially the maximum extension of the search radius and the value by which the radius is extended allows for control of clustering and agent type distribution of personal networks.

1 MOTIVATION

The generation of social networks is an important issue in agent-based modelling. The network structure might have considerable impact on certain processes like opinion formation [5], information exchange for problem solving [10], or advice [22]. Furthermore, [4] investigates the impact of network structure in a model of racial segregation and comes to the conclusion that the structure of the social network, and especially its relation to physical space, has significant effects on the results of social simulation.

Usually, simulations generate social networks according either to the small world algorithm proposed by [21] ([8; 9]) or to preferential attachment [1]. These methods focus on producing networks whose global, i.e. network level properties like average path length, clustering coefficient, and degree distribution are as similar as possible to empirically found values. However, these methods neglect local circumstances as well as actor properties and preferences and/or require global network knowledge during the generation. Whereas such aspects may be insignificant with respect to rather theoretical applications they might play a key role in many social science simulations, for instance in modelling for policy consulting.

Social networks are mostly characterised by what is often called the homophily principle. That is, people tend to build up relationships with others that are similar in some or many personal and socio-demographic attributes like age, gender, ethnic origin, educational background, or income. Thus, homophily narrows the people's social world in a fundamental way and influences their access to information, the way they are forming attitudes and the persons they meet. [11] distinguishes between baseline and inbreeding homophily. The former describes the phenomenon that people often live in surroundings with similar others. Consequently, the chance to spend time with that group and build up acquaintances is higher because of the composition of potential others. As a result, more trust occurs in such groups of similar people and network flows of information may increase. On the other hand, barriers between groups may exist which hinder information to spread [19]. The latter term describes the explicit tendency for persons to choose friends that have similar views, related occupations and like the same hobbies above the opportunity set.

[2] present an elaborated model based on social distance attachment that takes inbreeding homophily into account. The probability to link is derived from the sum of distances between individuals regarding each value of a vector representing the individuals' social coordinates. Resulting networks are compared with empirical data of the PGP (pretty good privacy) web of trust, and convincing similarity is obtained with respect to assortativity (i.e., the tendency that highly connected persons tend to have links to others with a high degree and vice versa) and a hierarchical community structure. However, the authors do not consider asymmetrical relationships. One way to accomplish directed networks is to define an individual's position in the social space for both in-going and out-going links.

[6] proposes a network generation method based on social circles [16]. Similar to [2] agents are located on a kind of social map according to certain, e.g. socio-demographic, properties. Whereas [2] proposes a city-block based distance measure (L_1) [6] applies an Euclidean based measure (L_2). Agents whose so-called reaches of a specific radius around their position on the map match each other's get connected. Again, this approach is not suitable for asymmetrical relationships. Furthermore, whereas it is possible to reflect agent specific ego network sizes by different reaches the two-dimensional map does not allow for placing agents according to more than two properties.

For his agent-based simulation [20] accounts for inbreeding homophily tendencies and connects agents according to their network preferences, i.e. the number of desired relationships and the liking either for similar or sometimes even dissimilar persons. The author further discriminates between normative, i.e. influencing, and informational ties. Finally, deviations are defined with respect to the number of relationships, the amount of correct relation types, and the number of desired similar and dissimilar ties. Agents then shall be connected in ways that minimise these deviations.

We propose a network generation process that takes into account baseline as well as inbreeding homophily. Since we build up a spatially explicit social simulation we are mainly interested in the spatial restrictions and opportunities actors face when they make up relationships. An actor may only connect to those others who are available within the boundaries he is agitating. For instance, the choice of network partners may vary

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whether someone lives in a dense urban environment with manifold others to choose from, or in a sparsely populated rural area.

2 OUR APPROACH

An important and comprehensive source of heterogeneity of people is their grouping according to sociological lifestyles. Because of societal liberalisation social norms based on social classes decay and individuals experience more autonomy. Lifestyles seek to capture perceivable patterns of behaviour, symbolic integration and underlying orientations as expressions of that autonomy. Lifestyles are thus meant to be a more relevant grouping of individuals and households [18]. We apply the Sinus-Milieus® [17] that are commonly used in commercial market research, but also in environmental research [14]. Sinus-Milieus® group individuals or households along the classical dimension of social status given by income and education, and supplement this grouping by a second dimension that reflects social value orientations like tradition, modernisation and re-orientation.

The empirical base for the results presented in this paper is a dataset of spatially referenced socio-demographic data of the target region of Northern Hesse located in the centre of Germany. Data originate from a 2007 survey by Microm® [12]. The geographical reference units are cells that comprise one to several hundred households depending on population density. For each of the cells we extract the number of households belonging to each of four different lifestyles: Leading lifestyles are characterised by the pursuit of prestige as well as wealth and occupation of leadership positions. Traditional lifestyles are often adopted by worker families that desire security and order. The mainstream strives for professional and societal establishment and harmonic circumstance, whereas a hedonistic lifestyle is characterised by the search for pleasure, sometimes with little resources, and often the denial of conventions.

In order to apply our network generation approach considering baseline and inbreeding homophily we first initialise an agent population such that the distribution of lifestyles among agents and the agents' location reflect the empirically observed spatial distribution of lifestyles. To do so we first determine the number of required representatives for each lifestyle in every data cell. Then, we initialise each agent as a representative for ten households of a specific lifestyle and place it normally randomly close to the respective cell in a GIS (see figure 1). The resulting population setup is empirically founded and provides spatial relationships between agents as well as lifestyle heterogeneity.

Since we are interested in processes of social influence we model relationships between agents as asymmetrical ties that are represented by directed links in a network. These links have their origin in the influencer and lead to the agent that is being influenced. Therefore, the in-degree of an agent's personal network (also referred to as ego network) specifies the number of network partners that influence that agent. Table 1 presents the lifestyle specific network preferences.

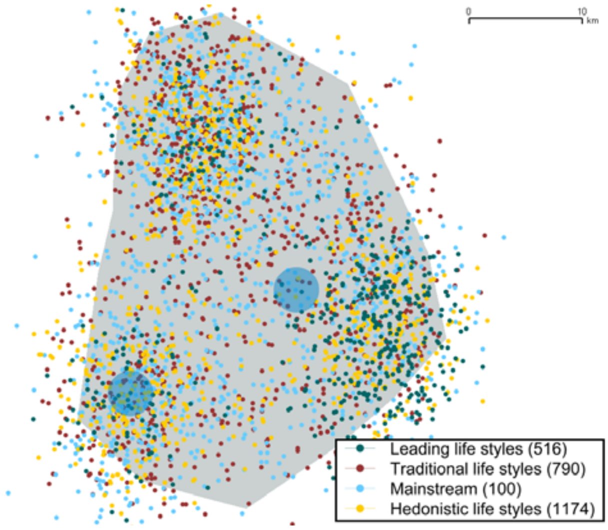

Figure 1: Points represent agent positions within the model region whereas colors specify the agent's lifestyle. Numbers in brackets are the amount of agents of that lifestyle. The total number of agents within the model region is 3480. Cumulations indicate three smaller cities. Blue shaded circles show a search radius of 2000m around an agent in rural area and an agent within a city.

	Leading	Traditional	Mainstream	Hedonistic
In-degree	15	5	5	10
p_rewire	0.2	0.05	0.1	0.2
p_links to				
Leading	0.8	0.0	0.0	0.2
Traditional	0.6	0.3	0.1	0.0
Main-stream	0.6	0.1	0.3	0.0
Hedonistic	0.5	0.0	0.0	0.5

Table 1: Expert rating of lifestyle network preferences. Whereas members of leading and hedonistic lifestyles have far reaching networks and thus are assigned a high rewiring probability, people of traditional lifestyles do not. Data is based on [15].

The network generation is divided into two parts, the establishment of local links and the rewiring process. Each single part is processed iteratively for all agents. As depicted in figure Figure 2 the first part starts with collecting and shuffling all agents within the current search radius which is initially given by START_SEARCH_RADIUS. For every potential partner that is not yet connected with the focal agent it is decided according to the lifestyle specific probability (see p_links in table 1) if it should be linked to the focal agent.

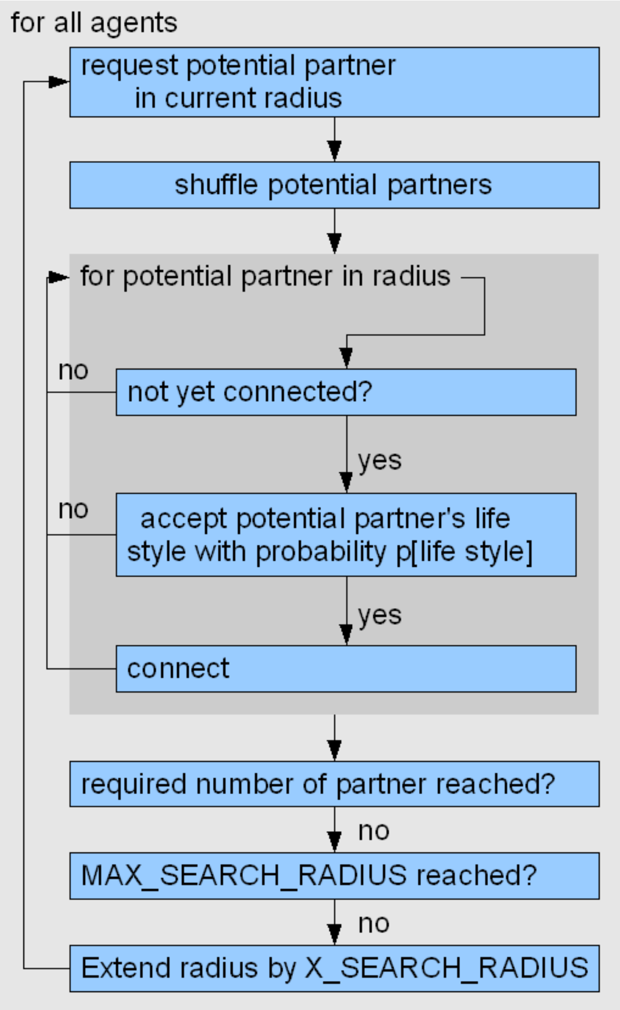

Figure 2: Course of local network generation (rewiring not included)

If the number of required network partners is not reached but all collected agents are treated, more agents are collected from around the focal agent within a current radius that is extended by X_SEARCH_RADIUS. This loop is repeated until either the number of required network partners is satisfied or maximum radius (MAX_SEARCH_RADIUS) is reached.

The approach to select surrounding agents as they come considers the local lifestyle composition and reflects baseline homophily. However, this way the algorithm accounts not only for groups of similar agents that stick together but also for opposite situations when one cannot establish connections to those people one would like to. Applying lifestyle specific preference probabilities when accepting or rejecting a potential network partner reflects inbreeding homophily finally.

After each agent is connected locally the global rewiring process takes place during the second part. For each agent and every existing local link, with probability P_REWIRE (see p_rewire in table 1) the link is rewired to a randomly chosen agent from the entire model region. The random target agent selection is repeated until the found agent is accepted according to the lifestyle specific preferences probabilities (p_links).

The emerging distant links result in the small world effect with high clustering and low average path lengths. On purpose the new partner's lifestyle needs not to be the same as that of the originally linked: The composition of network partners within direct surroundings is characterised by the local lifestyle distribution (baseline homophily) and therefore does not entirely reflect the focal agent's network partner preferences (p_links). Determining the lifestyle during rewiring anew may correct this lifestyle composition of network partners towards imbreeeding homophily and thus is desired.

3 RESULTS

We implemented our spatial agent-based model in Repast Simphony [7]. Data is exported to a database and processed by R [13; 3]. Results are averaged over five independent model runs with different random seed.

We compare the results of our proposed algorithm that takes baseline homophily into account with an ideal network builder and a small world generator [21]. The ideal network builder tracks the lifestyle of network partners and allows a link between the focal agent and a potential alter only if the focal agents has not yet built enough connections to other representatives of the alter’s lifestyle.

To evaluate the appropriateness of certain algorithm variations and parameter settings we introduce some quality measures. The deviation from preferred lifestyle distribution of partners (preference deviation) compares the desired personal network's lifestyle composition with the actual one. The measure sums up the deviation for each of the four lifestyles. The deviation from preferred in-degree to the actual number of influencing others is referenced to as in-degree deviation. Furthermore, we consider the average path length (average network distance of all node pairs in the network) and the global clustering coefficient, also known as transitivity index, which in our case is the number of all existing triples divided by the total number of triangles, i.e. potential triples.

It is important to note that the measures highly depend on the distribution of agents across the model region, especially with respect to lifestyles. Our model region as depicted in figure 1 is a rather rural area with three small cities. For agents in the centre of the area it will be quite hard to satisfy their links with respect to inbreeding homophily. This is especially true for people of leading lifestyles that occur very sparsely in the centre but like to connect predominantly to other people of a leading lifestyle.

There are some parameters to adjust the network's characteristics. Whereas the MAX_SEARCH_RADIUS defines the geographical area within which agents may search for partners, X_SEARCH_RADIUS denotes the value by which the search radius is extended in case the current radius is not far enough to fulfil the number of partners the agent desires. Furthermore, the rewiring probability influences the amount of rather distant links.

Figure 3 shows the network in-degree deviation as a function of MAX_SEARCH_RADIUS. The smaller the radius, the less space is given to fulfil the agents’ preferences regarding the lifestyle distribution of their social network. The algorithm considering baseline homophily yields lower deviations for larger radii since it allows connections to alteri that do not match the preferred lifestyle distribution. Of course, regarding network preference deviation the ideal network builder performs better since that is its purpose. As figure 4 clearly indicates, with increasing MAX_SEARCH_RADIUS the deviations can be reduced. Leading lifestyles improve only slightly since the overall number within the model region is limited. In terms of modelling realistic social networks a specific deviation is desired for certain lifestyles since it reflects social settings.

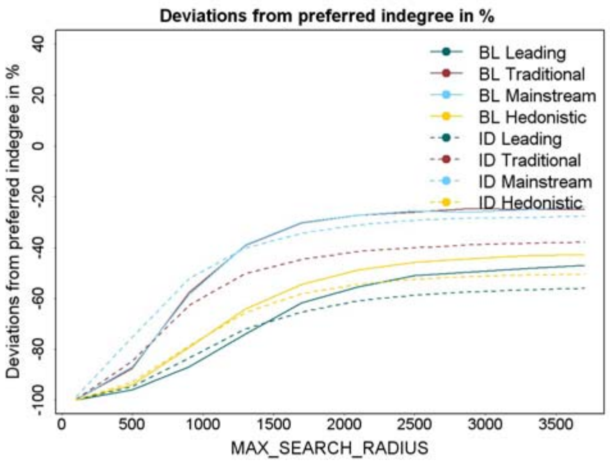

Figure 3: Percental Network in-degree deviation with raising MAX_SEARCH_RADIUS. Negative values indicate that actual degree is smaller than preferred. For smaller radii, the algorithm considering baseline homophily (BL – dashed lines) yields higher deviations from the preferred in-degree (number of influencers) than the ideal network builder (ID – solid lines).

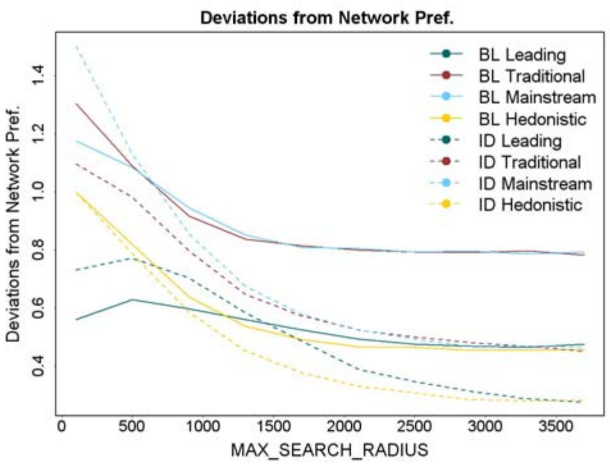

Figure 4: Deviations from milieu-specific network partner preferences with raising MAX_SEARCH_ RADIUS. Compared to the ideal network builder (ID-solid lines) the baseline algorithm (BL – dashed lines) results in higher deviations (apart from hedonists). With increasing MAX_SEARCH_RADIUS deviations become smaller.

As figure 5 shows, the average distance to a neighbour is considerably lower in networks from the proposed builder. Of course, this is due to the local search for neighbours the small world generator does not take into account. As the rewiring probability raises also the average distance increases.

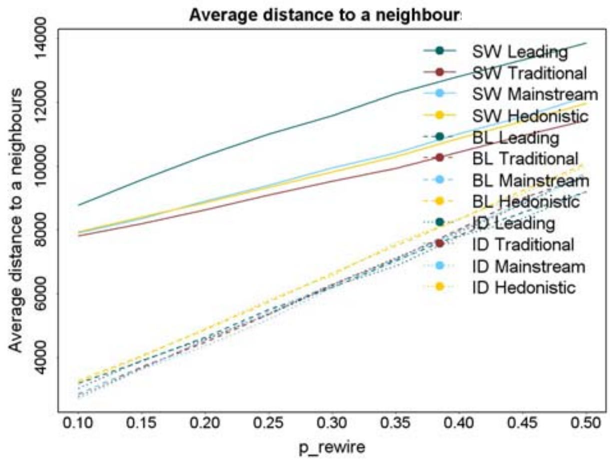

Figure 5: Average distance to a neighbour in meters. Since the small world generator does not explicitly consider spatial proximity, the distance is larger.

The rewiring probability relaxes the MAX_SEARCH_RADIUS in the way that it allows the agents to choose the more agents deliberatively within the entire simulation area the higher the probability is. Furthermore, it is in particular responsible for the small world properties and thus affects the average path length and the clustering coefficient. The global clustering coefficient gives an important hint towards the empirical foundation of the proposed network generation algorithm. The higher the amount of local links that are rewired globally the lower is the clustering coefficient (see figure 6) length and lower is the average path (see figure 7) [21].

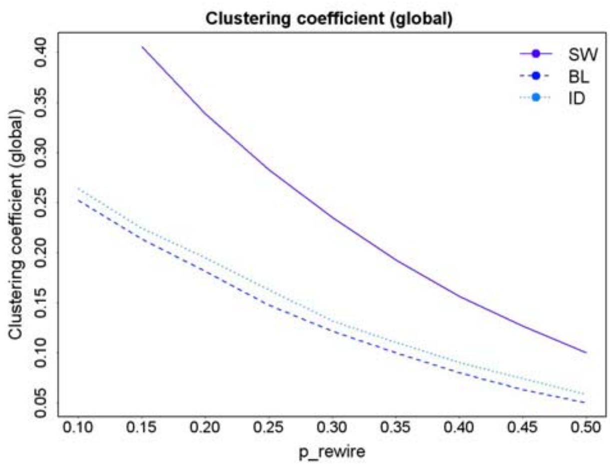

Figure 6: The global clustering coefficient drops strongly when more and more local links are globally rewired. The small world generator yields a much higher clustering.

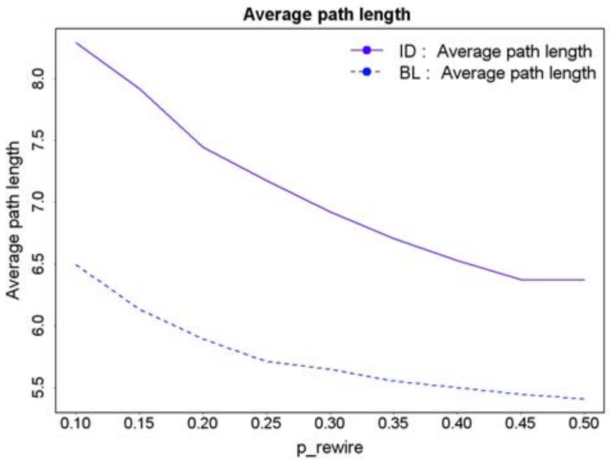

Figure 7: The Average path length decreases along with the establishment of more distant relationships.

As figure 8 indicates, variations in the rewiring probability have also a minor impact on the network preference deviations. Whereas for the proposed algorithm deviations decrease because rewiring guarantees a partner of desired lifestyle, network produced by the ideal network builder do not benefit from rewiring. That is because the target agent is not guaranteed to be of the desired lifestyle.

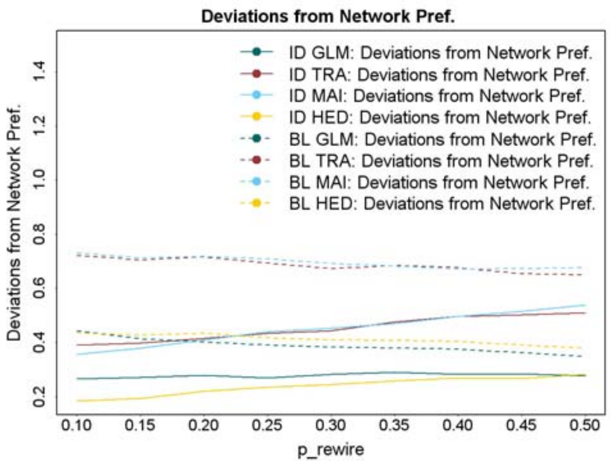

Figure 8: For the baseline homophily considering approach, deviations from preferred lifestyle distribution of network partners decrease with increasing rewiring probability since rewiring supports partners of desired lifestyle.

Figure 9 shows the effect of altering the X_SEARCH_ RADIUS, that is the radius by which the search radius is extended in case the number of required partners can not be fulfilled, has on the clustering coefficient. If the search radius is raised slowly, agents are forced to build up connections with nearby agents which supports local clustering. However, since a smaller search radius reduces the opportunity set, the network preferences deviation is lower for higher values for X_SEARCH_RADIUS.

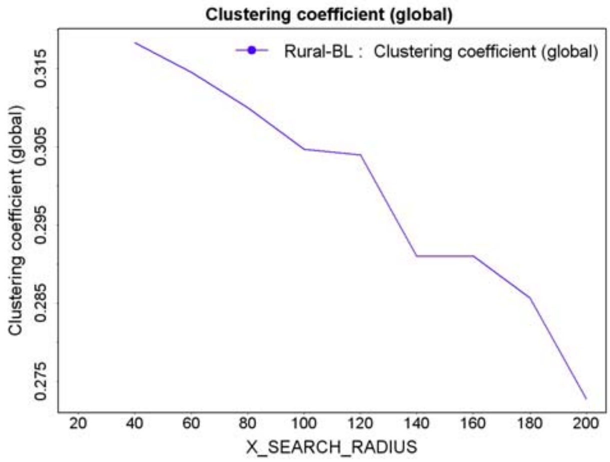

Figure 9: Raising X_SEARCH_RADIUS when the initial search radius is rather small (20m). The clustering coefficient is higher for small values of X_SEARCH_ RADIUS when agents are forced to build up rather local connections.

Finally, we investigate the impact of the baseline homophily considering approach on the out-degree distribution, i.e. the number of network partner a focal agent may influence. Compared to the ideal network builder agents are assigned more outgoing relationships. The reason is that the baseline homophily concept is less strict in the selection of alteri. Leading lifestyles (dark green line) are especially central in the network (figure 10).

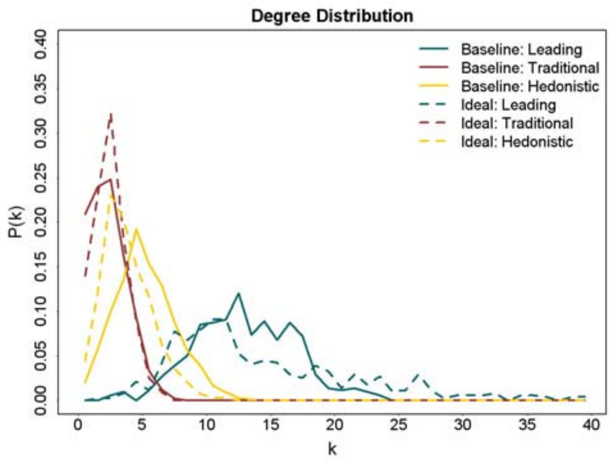

Figure 10: Distribution of out-degree for MAX_SEARCH_RADIUS of 2500m and X_SEARCH_ RADIUS of 100m. Since the baseline algorithm is more flexible in assigning partners degree distributions are shifted to the right.

In comparing the baseline homophily considering network generator with a small world generator we find that the latter yields somewhat smoother network properties (e.g., see clustering coefficient in figure 6). However, taking the principles of baseline homophily into account might question the realism of that widespread network generator's foundation. As figure 11 shows, the proposed network generator results in moderate assortative mixing, due to local restriction in partner selection.

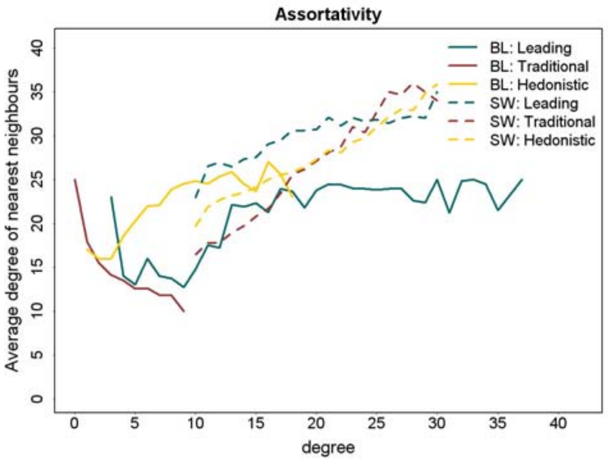

Figure 11: The average degree of nearest neighbours as a function of degree shows moderate assortative mixing (MAX_SEARCH_RADIUS: 3000m, rewiring of 0.1, X_SEARCH_RADIUS: 500m).

4 DISCUSSION

We proposed a simple but powerful approach to generate social networks for spatial agent based models. It seeks to reflect realistic, natural settings of the model region and also shows desired, empirically grounded network properties like short average path length, considerable clustering, and moderate assortativity. Therefore, we describe an alternative to the widespread small world algorithm which lacks realistic groundings with respect to local interactions.

The resulting network may be adjusted by setting the MAX_SEARCH_RADIUS (to set the moving radius of actors which might differ considerably from area to area and from life style to life style), the X_SEARCH_RADIUS (the radius by which the search radius is extended as long as more agents are required to choose from and MAX_SEARCH_RADIUS is not reached), and the P_REWIRE (to account for network parts that outreach the local region). Furthermore, the lifestyle preferences of each agent type may be adjusted. The MAX_SEARCH_RADIUS provides an adequate regulator to adjust milieu-specific radii of action and thus reduce the network preference deviation while preserving clustering. X_SEARCH_RADIUS helps to control the clustering coefficient, while p_rewire has an impact on the average path length.

Probably the greatest challenge in modelling social networks is gaining adequate empirical data about the relations modelled actors have. An advantage of our approach is that every parameter could be more or less empirically measured. For instance, the MAX_SEARCH_RADIUS is determined by the area a person normally agitates within. The network size and preferences regarding life styles could be gained by analysing personal networks of an adequate amount of representatives of each life style. However, since such explorations are quite demanding and expensive one most often has to guess values from experience or consult experts in the field.

In the future we seek to further explore the parameter space of the network generation in order to predict the properties of resulting networks more thoroughly. Emphasis is placed on the interplay between the mentioned parameters. For instance, both the rewiring probability and X_SEARCH:_RADIUS have an impact on the global clustering coefficient. Besides it is worth to explore heterogeneous, lifestyle specific parameters.

A possible extension is to allow agents to start their search within a specific annulus around their home coordinates and then broaden it simultaneously to the inner and the outer area. This would account for people that refuse to make connections within their direct neighbourhood. Furthermore, extensions in the direction of incorporating geographical and social distance as proposed by [2] is expected to be fruitful.

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Selected Models for Agent-based Simulation of Social Networks

Federico Bergenti[1] and **Enrico Franchi** and **Agostino Poggi**[2]

Abstract. In this paper we review some classic models of the static structure of complex networks with the objective of finding a good model for simulating a large-scale, technology-enabled social network. First, we outline the basic properties that characterise such social networks with respect to other networks. Then, we briefly discuss some classic network models and their properties, and, finally, we match the properties of the models against the characterising features of social networks. In the end, we present an agent-based framework we are building to experiment with network generation models.

1 INTRODUCTION

The notable interest of today's research on social networks is largely justified by their adoption as a unifying metaphor in very relevant Web-centric services like Facebook and many others. The inherently large scale of such services calls for automated techniques capable of promoting their potentials to unforeseen levels in terms of offered functionality and performance. Such automated techniques are still far from real-world practice because the impact of a novel algorithm (e.g., a friendship-discovery algorithm) cannot be easily assessed. This is the reason why we need effective tools to study, experiment and validate innovative techniques capable of providing concrete evaluation on the net results of the introduction of a novel proposal into a social network. *Agent-based simulation* is very helpful to this respect because it provides solid approaches for testing new ideas *in silico* before trying to put them into practice.

Agent-based simulation is now a consolidated field of research, and likewise it is the application of its results to social networks. However, the use of agent-based simulation for a *large-scale, technology-enabled social network* still lacks an accepted, formal model of network meant to generate suitable (artificial) networks for running experiments. In this paper we review some classic network models and briefly show their characteristics from the point of view of generating networks for running simulated experiments. We rank such models and we identify the model best suited for our purposes.

This paper is organised as follows: in next section we introduce some metrics that are generally applicable to all networks and that we will use to characterise social networks. Such metrics are very common in network theory and we review them just to provide a common notation and a precise understanding of concepts. In Section 3 we survey some of the most classic network models and we briefly present some results on the metrics that we introduced in the previous section. In Section 4 we provide a characterisation of large-scale, technology-driven social networks using the identified metrics and in Section 5 we review the discussed network models and we show how well each model can be used to produce networks that exhibit the structure of a social network. In Section 6 we introduce some preliminary work on agent-based simulation, based on the more successful generative models. Finally, in Section 7 we draw some conclusions and sketch a possible line of work.

2 NETWORKS METRICS AND MEASURES

In this section we briefly review some classic metrics which give insight on the network structure. The metrics we have chosen are some of the most widely studied in network analysis and together they give a rough idea of the structure of a network; we also outline some correlation in the chosen metrics. In this paper we do not deal with advanced analysis techniques, such as community or cluster detection, since most of the papers we discuss in the following sections do not deal with them at all as well and consequently the results could not be compared.

In this paper, the properties taken into account are: (*i*) average shortest path length/diameter; (*ii*) clustering coefficient; (*iii*) degree distribution; (*iv*) assortativity coefficient; (*v*) navigability.

Before introducing the metrics, we introduce the notation we use. With A we refer to the adjacency matrix of the analysed network.

If u is a node in a directed network, (*i*) the in-degree k_u^{in} is the number of incoming edges $\sum_i A_{iu}$; (*ii*) the out-degree k_u^{out} is the number of outgoing edges $\sum_j A_{uj}$; (*iii*) k_u is the sum of the in-degree and the out-degree. For undirected networks, the degree k_u is the total number of edges of u.

With $\langle\cdot\rangle$ we refer to the expected value of a quantity. We usually omit the elements participating in the sum, when it is clear from the context. For example we simply write $\langle k\rangle$ instead of $\langle k_i\rangle_{i\in G}$ to refer to the average degree of the nodes in the network.

In order to compare directed and undirected networks, we ensure that the directed networks are highly symmetrical. The measure of how symmetric is an undirected network is called *reciprocity* (or simply *symmetry*). If m is the number of edges in the network and A is the adjacency matrix of the network, then the reciprocity coefficient is $\frac{1}{m}\sum_{ij} A_{ij}A_{ji}$. The coefficient is trivially 1 for undirected networks. The social networks analysed all show a high symmetry coefficient, and we will not deal with it further.

Classic metrics in network analysis are the *average shortest path length* (ASPL), the *characteristic path length* (CPL) and the *diameter*. Let v and v' be two vertices in the network, then $L(v, v')$ is the length of the shortest path connecting v to v' (also called *geodesic path*). The closeness L_i of a node i is the mean of the geodesic distance between i and all the vertices reachable from it, that is to say:

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$L_i = \langle L(i,j) \rangle_j$. The shortest path length and the characteristic path length are the mean and the median value of all the L_i respectively. The diameter is the longest geodesic path.

In the context of network analysis the diameter, the CPL and the ASPL) are said to be *short* if they depend logarithmically from the number of nodes in the network.

A link $e = (u, v)$ is a *shortcut* (or *long-range* link) in the network $G = (V, E)$ if $L_{G'}(u, v) \gg 1$ where $G' = (V, E \setminus \{e\})$; otherwise it is a *local* or *short-range link*.

Another very important metric in the context of social networks is the *clustering coefficient* C, which is the mean of all the *local clustering coefficients* C_i, where C_i is is the fraction of pairs of neighbours of i which are also connected [38]. A different and non equivalent definition is given in [32], where the clustering coefficient is defined as the fraction of paths (u, v, w) of length two in a network $G = (V, E)$ for which $\{(u, v), (v, w), (w, u)\} \subseteq E$ holds.

The *degree distribution* of a network is simply the frequency distribution of vertex degrees. p_k is the fraction of vertices in the network with degree k. If the network is undirected, then there are two different degree distributions: the in-degree distribution and the out-degree distribution. Although in principle they can be very different, in practice in the examined contexts they are very similar (because the analysed networks are highly symmetric) and consequently we simply refer to the degree distribution. We say that a network has a power-law degree distribution with exponent γ if $p_k \propto k^{-\gamma}$. γ is also called the *scaling exponent* and networks whose degree distribution is a power-law are usually called scale-free, because of the scale invariance property of power-law, i.e., if f is a power-law with exponent γ, $f(c \cdot x) = a(cx)^\gamma = c^\gamma \cdot ax^\gamma = c^\gamma f(x) \propto f(x)$.

Scale-free networks are of particular interest to us since: (*i*) essentially all the networks analysed in Section 4 are scale-free; (*ii*) in [10] it has been proved that a wide category of scale-free networks have short diameter *because* they are scale free. To be more precise, it has been proved that if the network has a power-law degree distribution of exponent γ, the diameter d is

$$d = \begin{cases} \log \log N & \gamma \in (2, 3) \\ \log N / \log \log N & \gamma = 3 \\ \log N & \gamma > 3 \end{cases} \tag{1}$$

Another very important property related to the degree distribution is the *assortativity coefficient* r, which is basically the Pearson product-moment correlation coefficient of degree between pairs of linked nodes. A network is *assortative* if it has positive assortativity coefficient. The definition of r is:

$$r = \frac{\langle k_i k_j \rangle - \langle k_i \rangle \langle k_j \rangle}{\sqrt{\left(\langle k_i^2 \rangle - \langle k_i \rangle^2 \right) \left(\langle k_j^2 \rangle - \langle k_j \rangle^2 \right)}} \tag{2}$$

The last property of social networks we take into account is the *navigability*. We say that a network is navigable if it exists a simple decentralised algorithm that is able to deliver a message to any node, starting from any node, in polylogarithmic number of steps. With "simple" we mean that each node passes the message to a single neighbour using some ranking function to decide which one. The ranking function must not encompass global knowledge of the long-range links. The *delivery time* of an algorithm is the expected number of steps required to reach the target, randomly choosing the start and the end node.

3 MODELS FOR SIMULATED NETWORKS

In this section we show some models to generate random graphs which we would like to use to simulate social networks.

The first and still most studied model of random graphs is the *Erdős-Rényi model* (ER) $G(n, p)$ [15, 33]. $G(n, p)$ is a probability distribution over the set of all graphs with n nodes. The p parameter indicates that an edge is placed between any given pair of nodes with probability p. Consequently, (*i*) each individual graph is chosen with probability $p^m (1-p)^{\binom{n}{2} - m}$; (*ii*) the expected value of the number of edges is $\langle m \rangle = \binom{n}{2} p$; (*iii*) the expected mean degree is $\langle k \rangle = (n-1)p$; (*iv*) the expected diameter is $\log n$; (*v*) the degree distribution tends to a Poisson distribution for large n; (*vi*) the clustering coefficient is given by $C = \langle k \rangle / (n-1)$.

Another very important model is the *Strogatz-Watts model* (SW) introduced in [38]. The model starts with a closed linear structure where each node is connected with κ neighbours and then the local connections are rewired to remote nodes with probability p. The rewired connections are usually *shortcuts*. p is a parameter governing the transition from the very regular lattice ($p = 0$, no rewiring) to $G(n, \hat{p})$, where $\hat{p} = n\kappa / 2\binom{n}{2}$. In this model, the mean degree is exactly κ. The other metrics are rather hard to derive for this model, however, a minor variant of this model has been analysed in [9, 31, 35]. In this variant the shortcuts are added without removing the local connection. To be more precise, for each link in the lattice a shortcut is added with probability p. Consequently the average number σ of long-range links each node gains is $p\kappa$ according to the distribution:

$$p_\sigma = e^{-p\kappa} \frac{(p\kappa)^\sigma}{\sigma!} \tag{3}$$

The average shortest path length is logarithmic with the size of the network, at least for large networks and the clustering coefficient is:

$$C = \frac{3(\kappa - 2)}{4(\kappa - 1) + 8\kappa p + 4\kappa p^2} \tag{4}$$

In [20] a model using somewhat similar ideas although starting from a different regular structure is presented. In the paper the starting structure is a 2D grid where each node is connected to its neighbours and shortcuts are added. A shortcut between node u and v is added with probability proportional to $d(u, v)^{-\alpha}$, where d is the Manhattan distance between nodes u and v in the grid, α is a parameter. This model has been extended to use a k-dimensional mesh as a starting structure. In [36] there are several results on the diameter of such networks: (*i*) if $\alpha \in [0, k]$ then the diameter is $\Theta(\log n)$; (*ii*) if $\alpha \in (k, 2k)$ then the diameter is polylogarithmic; (*iii*) if $\alpha > 2k$ then the diameter is polynomial; (*iv*) for $\alpha = 2l$ the diameter length is still an open problem. The clustering coefficient is naturally quite high (coming from a very regular structure).

However, the most interesting property of this model is that the delivery time T of any decentralised algorithm in the 2D grid based model is:

$$\mathrm{T} = \begin{cases} \Omega\left(n^{(2-\alpha)/3}\right) & \text{if } 0 \leq \alpha < 2 \\ \Theta\left(\log^2 n\right) & \text{if } \alpha = 2 \\ \Omega\left(n^{(\alpha-2)/(\alpha-1)}\right) & \text{if } \alpha > 2 \end{cases} \tag{5}$$

Similar results have been given for the k-dimensional grid models, where $\alpha = 2$ is substituted by $\alpha = k$.

In [21] the group model is presented. The model is not really a model for generating a network, however it can be used to make any network navigable adding some shortcuts. The process starts creating a finite family $\mathcal{S}$ over the set of nodes V satisfying the following conditions for some $\lambda \in (0,1)$ and $\beta > 1$: (*i*) $V \in \mathcal{S}$; (*ii*) $S_i \in \mathcal{S}$ and $|S_i| \geq 2$ such that $v \in S_i$, then there exists $S_j \in \mathcal{S}$ such that $S_j \subset S_i$ and $|S_j| \geq \min(\lambda g, g-1)$; (*iii*) if *a*) $S_i, S_j, S_k \ldots$ are in $\mathcal{S}$, *b*) have size at most q and *c*) v is in their intersection, then their union has size at most $q\beta$.

The sets in $\mathcal{S}$ are called groups. These conditions hold taking as the groups the balls determined by the Manhattan distance on the k-dimensional grid, for example.

Let $q(u,v)$ be the size of the minimum group in $\mathcal{S}$ containing both u and v. A *group-based model with structure $\mathcal{S}$, exponent α and out-degree m* is a network where for each node u a shortcut to v has been added with probability proportional to $f(q(u,v))$ and $f(x) \asymp x^\alpha$ and the process is repeated for m times.

In [21, 22] Kleinberg proved that for a network (V,E), given an arbitrary finite family $\mathcal{S}$ of sets over V satisfying properties (*i*), (*ii*) and (*iii*), there is a decentralised algorithm with polylogarithmic delivery time in the group-based model with structure $\mathcal{S}$, exponent $\alpha = 1$ and out-degree $m = c\log^2 n$ for a sufficiently large constant c. He also gave negative results for the existence of such algorithm for both $\alpha < 1$ and $\alpha > 1$. The group-based model is a meta-model. Most metrics depends on the underlying network structure; however, we can expect a reduction of the diameter due to the added links.

Another important meta-model is described in [13]: the authors show a procedure to turn a wide variety of network topology in navigable small-worlds. As for the group model, most metrics depend on the original network topology.

Popular models to generate scale-free networks are the ones based on preferential attachment (PA), where links are added more often to nodes with higher degree. In this family of methods the network is generated through multiple steps. At each step some edges and links are added or removed according to some rules that vary from model to model. A popular model of this family is the *Barabási-Albert model* (BA) described in [5]. The BA model starts with n_0 nodes and no edges. At each step a new node with m random links is added. The m links are directed towards node with probability proportional to their degree. The BA model generates only networks whose degree distribution is a power-law with exponent 3, on the other hand other preferential-attachment models yield scale-free networks with any exponent.

In [10] it has been proved that being scale-free with a degree $\gamma > 2$ implies having a short (polylogarithmic) diameter. Considering that the diameter is an upper bound of the geodesic paths in the network, the results also bounds the characteristic path length. No such results are available on navigability, it is however reasonable to use the meta-models to add such a property.

The basic PA process or the BA model do not generate networks with high clustering coefficient. For example, it has been empirically found that for a BA graph $C \sim n^{-0.75}$. No analytical method to compute C for the BA model is known [4].

A method to increase the clustering coefficient is mingling PA steps with triadic closure (TC) steps. During the TC step, if a link between u and v was added in the PA step, then it is added also a link between u and a random neighbour w of v. This model yields networks with high clustering coefficient and has been extensively studied in [18, 37].

Another model in the family of PA models is the *biased preferential attachment* described in [23]. The set of nodes V is partitioned in three sets P, I and L. At each new step (*i*) a new node is added to the network and is assigned to one of the three sets according to a distribution of probability p; (*ii*) $\epsilon > 0$ edges are added to the network. Essentially both p and ϵ are parameters that can be tuned; there is also a third parameter γ. D^β is a probability distribution such that for each node u:

$$D_u^\beta \propto \begin{cases} (\beta+1)\cdot(k_u+1) & u \in L \\ k_u+1 & u \in I \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

The ϵ edges are added according the following rule: for each edge (u,v), u is chosen with distribution D_0 and (*i*) if $u \in I$, v is a new node and is assigned to P; (*ii*) if $u \in L$, v is chosen according to D^γ. In [23] there are no analytical results about the network metrics. However, the authors claim they were able to reproduce parameters they measured in two real social networks (Yahoo 360 and Flickr). Consequently we expect that, at least for some choice of parameters, the methods yields a network with high clustering coefficient, short diameter and power-law degree distribution.

The last model we review is called *transitive linking* [11]. The model is somewhat similar to the PA model with the addition of the TC step. However, the model also accounts for the possibility that nodes leave the network. In every step of the method two things occur: (*i*) a random node is chosen, and it introduces two other nodes that are linked to it, resulting in a new link (this is the transitive linking, in short TL); (*ii*) with probability p a node is chosen and removed from the network and its edges are removed as well and replaced with another node with one random edge. If the node chosen in (*i*) does not have two edges, then it introduces himself to another random node. The parameter p dictates how often someone is removed from the social network and is assumed to be much smaller than 1.

When $p \ll 1$ the TL dominates the process and the degree distribution is essentially a power-law with a cutoff for larger k, as nodes have finite lifetime. For larger values of p the two different process concur to form an exponential degree distribution, while for $p \approx 1$ the degree distribution is essentially Poisson distribution. For $p \ll 1$ the clustering coefficient is rather large and can be determined with the relation $1 - C = p(\langle k \rangle - 1)$; as p decreases $\langle k \rangle$ grows. For example, for $p = 0.01$, $\langle k \rangle = 49.1$ and $C = 0.52$. The authors also calculated that:

$$\text{ASPL} \approx \frac{\log(n/\langle k \rangle)}{\log\left(\frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}\right)} + 1 \tag{7}$$

4 ANALYSIS OF REAL-WORLD SOCIAL NETWORKS

In the early studies on social networks, the first step was the long manual gathering of data regarding the social network itself, using interviews or other ad-hoc methods. Consequently the social networks taken into account were relatively small and biases could be introduced by the sampling method.

With the widespread adoption of *social networking systems* by huge amount of people, it became possible to study social networks of unprecedented size. In this section we review a number of papers analysing different *online social networks* (OSN).

In Table 1 we have gathered metrics from some papers where existing social networks have been measured [1, 2, 3, 30]. When in the original paper a datum is missing, we placed "n.a." (not available) in the table. Moreover, some papers reported distinct in-degree and out-degree distributions: we used the notation $\gamma_{\text{out}}/\gamma_{\text{in}}$. In [3] the researchers found out that Cyworld has two different scaling regions, one with $\gamma = 4$ and one with $\gamma = 1$, the crossover occurs between $k = 10^3$ and $k = 10^4$. Consequently, in the table we reported γ for Cyworld as $(4; 1)$.

Table 1. Basic metrics for a selection of online social networks

Online Social Network	Refs.	Users	Links	% of V	k	C	CPL	d	γ	r
Club Nexus	[1, 2]	2496	10119	100%	8.2	0.17	4	13	*n.a.*	*n.a.*
Cyworld	[3]	12 M	191 M	100%	31.6	0.16	3.2	16	(4; 1)	-0.13
Cyworld Testimonial	[3]	92 K	0.7 M	0.77%	15.3	0.32	7.2	*n.a.*	*n.a.*	0.43
Orkut	[3]	100 K	1.5 M	0.3%	30.2	0.3	3.8	*n.a.*	3.7	0.31
Orkut	[30]	3 M	223 M	11.3%	106.1	0.171	4.25	9	1.50	0.072
Flickr	[30]	1,8 M	22 M	26.9%	12.24	0.313	5.67	27	1.74/1.78	0.202
Live Journal	[30]	5 M	77 M	95,4%	16.97	0.330	5.88	20	1.59/1.65	0.179
Youtube	[30]	1,1 M	5 M	*n.a.*	4.29	0.136	5.10	21	1.63/1.99	-0.033

While some researchers had access to the full body of data from the analysed online social network, most of them had still to resort to sampling techniques and consequently we reported the percentage of the social network they claim they have sampled. For further details on the sampling techniques we refer to the original papers.

In principle, sampling can introduce the same biases typical of earlier studies; nonetheless, the greatly increased amount of data available is a positive factor in its own right. The biases may be the reason why different studies on the same OSN find different values on the same metrics. However, the general trends are confirmed by all the studies we reviewed, and the general structure of online social networks appears not unlike that of offline social networks.

In fact, sociologist have known since a long time that social networks are highly clustered and OSNs show high clustering as well (e.g., in [17]). Like offline social networks, OSNs have a relatively high clustering coefficient C, orders of magnitude higher than that of random graphs. The actual value of the coefficient exhibits a large variability. Moreover it appears that this coefficient varies much for the *same* network depending on which data was available and how the researchers obtained it.

For example in [3] the measured clustering coefficient for Orkut is 0.31, while in [30] it is 0.17. Usually clustering coefficient is not uniform for nodes of different degree. For example in Cyworld [3] the nodes with degree $k < 500$ have a high local clustering coefficient, while friends of nodes with higher degree are not tightly clustered. This contributes to the relatively small global clustering coefficient.

Social networks have short characteristic path length and online social networks do not deviate, confirming the intuition that they actually are small-worlds; measured values of CPL vary between 4 and 6, which is consistent with expected values for small-world networks of comparable size [2, 30]. It is also interesting that studies taking into account the evolution of such social networks point out that the CPL varies in time: typically there is a period when the distance between users increases (which happens when many new users join) and then when the network becomes more dense the CPL and diameter fall [3, 23, 24, 25].

All the analysed OSNs present a power-law degree distribution; however, the coefficient differ greatly and some OSNs have the coefficient γ of the power-law smaller than 2 [2, 3, 23, 30]. It should be noted that while some of the considered networks are undirected, links have a very high level of symmetry and in-degree and out-degree distributions are very similar, most nodes in-degree and out-degree differ less than 20% of their value [30].

It is usually believed that human social networks are assortative [34]. However, this may not always be the case, some OSN have a negative assortativity coefficient. For example, Cyworld has a negative assortativity coefficient. In fact, the metrics of the social network depends much on the meaning the participants give to the links: a subgraph of Cyworld using a stronger notion of friendship shows a positive assortativity coefficient, reverting the trend of the complete Cyworld network. Although the assortativity coefficient is a very important property for a network, according to the present results, we believe that its positiveness should not be regarded as an absolute property of OSNs.

Club Nexus [1] is the only online social network among those reviewed so far for which experiments on navigability have been performed. However, the network proved to be too sparse to be successfully navigable. In fact the notion of "friend" implied in the social network was quite too strong with respect to the notion of "acquaintance" used in [12, 29] or even in other online social networks analysed in [1]. It is however generally believed that social networks should exhibit this property, considering the experiments in [29, 12]. We should notice that in [1] it is not proved that the students of Club Nexus were not able to route messages, but that greedy algorithms, using data provided from the OSN were not able to do it. We would also like to point out that for Club Nexus social network the expected average degree $\langle k \rangle = 8.2$, which is a value much lower than that reported in the other social networks.

Another important experiment regarding navigability of social networks has been performed in [28]. The OSN taken into account is LiveJournal, mainly because many users provide information on their geographical position and the authors wanted to investigate importance of geography in the distribution of shortcuts, which are the ones mainly responsible for the short paths and consequently for short delivery times. The idea that geography is an important factor had already been studied in [19].

The authors have experimentally proved using simulation that the network is navigable; however, they also discovered that the network does not have the long-range links distributed according to Kleinberg's claims, i.e., with probability proportional to the square of the distance, as the network has been placed on a 2D mesh.

However, they argue that the right heuristics for the greedy algorithm should not be the plain geographical distance, but the ranking function $\hat{r}(u, v) = |\{w : d(u, w) \leq d(u, v)\}|$. Using $\hat{r}$ as the ranking function, the probability that u is linked with v is proportional to $\hat{r}(u, v)^{-1}$.

5 ANALYSIS OF SIMULATED NETWORK MODELS

All the methods presented in Section 3 fail to catch some aspect of real world social networks, especially considering how different is the structure of the various OSNs we reviewed in Section 4.

It is not surprising that the *Erdős-Rényi model* fails to describe social networks: people do not establish relationships purely by chance, regardless of affinity and geographical distance. The degree distribution also deviates from that of the networks analysed in Section 4: they have a power-law degree distribution instead of a Poisson distribution. Moreover, in social processes, the average number of connections a person has does not depend on the size of the network [14]. Consequently if we used the ER model with a constant number of connections, $C \to 0$ as $n \to \infty$.

The SW model was introduced to cope with the shortcomings of the *Erdős-Rényi model*, in particular the very low clustering coefficient. In the SW model for a very large interval of p values the resulting graph is both highly clustered (like the starting lattice) and shows short characteristic path length like ER random graphs.

However, the degree distribution that can be derived from Equation (3) (see [33] for mode details) is very different from the power-laws found in real social networks, which usually have right skewed degree distributions [34]. Another shortcoming of this model is that it is not navigable, as proved in [20].

The 2D grid, the multi-dimensional grids and the group based models proposed in [20, 21, 22] are all navigable. However, the author is well aware that "*the full range of factors that contribute to the observed structure* (referring to real world networks) *will be too intricate to be fully captured by any simple model*" [22]. In particular, these models seem too unrealistic and too far from reasonable network formation processes to be used for network simulations, as real social network are not based on extremely regular topologies [24].

Classic preferential attachment models fail to yield highly clustered graphs in most their variants and are consequently unsuitable to model a social network. Indeed, the models have been developed to explain the metrics of citation networks. However, many variations on the original model which have a rather high clustering coefficient have been proposed in order to explicitly model social networks.

For example, the *biased preferential attachment* model described in [23] was built from the ground to reproduce some metrics of real world social networks. In fact, the model does not only yield a static social network with realistic properties but the whole dynamic generation process mimics the ones if the OSN the authors analysed.

The transitive linking model also looks very promising. People is far more likely to make friends with friend of friends [17]. The importance of adding explicit triadic closure steps has also been proved in [24], where the authors showed that regular PA, without steps adding local links between friends of friends failed to model real social networks. The other metrics are also compatible with the experimental data of Section 4.

In fact, we have chosen few metrics in order to simplify the analysis. The clustering coefficient is a coarse measure of the structure of the network. Community detection algorithms could give further insights on the fine-grained structure of social networks. Moreover, from studies such as [3, 23, 24, 26] it is clear that OSN are constituted by distinct areas with very different structure and we believe that this should be taken into account. In the present study we are only concerned with the metrics of the final network, with little focus on the metrics along the process itself, while in [23, 24, 26] the authors deal with processes which reproduce OSN metrics during the whole formation process. We decided to consider only the static analysis of the resulting network because: (*i*) many models are not meant as proper processes, since what looks like a process is in fact only an algorithmic description; (*ii*) it is particularly difficult to sample large social networks over long periods of time and consequently there is less data on the issue. Moreover, we are mostly interested in performing simulations on the final network. Nonetheless, it is worth noting how processes which are inspired by actual human behaviour [11, 18, 23, 37] are some of the most promising models.

6 SIMULATION MODELS IMPLEMENTATION AND EXPERIMENTATION

The two most promising models among the ones we analysed have a common pattern: when an edge has to be added to the network (*i*) a node is randomly selected; (*ii*) the selected node "chooses" the other end of the link.

Agent-based simulation is extremely well suited for these kind of problems. A controller agent selects the agent(s) that are going to add a link and then each of these agents chooses the other end of the link. All the logic of the selecting the starting nodes is embedded in the controller agent and the logic of choosing the edge endpoint is in the agents.

We have started experimentation with such a system and we found that the flexibility of this model is significant. For example, once the regular PA model is implemented, adding also the biased preferential attachment model [23] is a matter of few lines of code. In general, models where there are different "kind" of nodes become easily expressible, to the point that each node can in principle be different from all the other agents.

Another variation we are exploring and which would be troublesome for analytic models is that the "receiver" of the link could refuse the link; this makes even more sense in the context of individual agent preferences and is in general part of the very agent model.

Models based on PA strategies have distinct ages: steps are performed sequentially. An agent-based simulation allows to explore network generation algorithms with unusual time patterns, e.g., where each agent can independently activate with a given probability p at any time or where agents have different sociality, which means that agents choose how many edges to activate.

In order to investigate these topics, we started an experimentation using *HDS* [6], *Heterogeneous Distributed System*, that is a software framework that aims at simplifying the realisation of distributed applications by merging the client-server and the peer-to-peer paradigms and by implementing all the interactions among all the software entities of a system through the exchange of messages. This software framework allows the realisation of systems based on two types of software entities, actors and servers, that can be distributed on a (heterogeneous) network of computational nodes (from now called runtime nodes). Actors have their own thread of execution and perform tasks interacting, if necessary, with other software entities through synchronous and asynchronous messages. Servers perform tasks on request of other actors. HDS is implemented using the Java language and takes advantage of preexistent Java software libraries and solutions for managing concurrency and distribution.

HDS has been already used for implementing some agent based applications [6, 7] and provides two different ways for the deployment of actors and servers that allow either: (*i*) to assign a thread to each actor and server; or (*ii*) to share a thread among a set of actors and servers. Therefore, HDS can be easily used for simulating social networks where individuals are represented by agents implemented on the top of HDS actors. Moreover, with HDS it is possible to take advantage of the typical interaction protocols used by FIPA compliant agent based systems [16]. Finally, the simulation size is scalable by both: (*i*) distributing the agents on a set of computational nodes; and (*ii*) managing the execution of the agents of a computational node through a single thread.

Another HDS feature we used extensively is possibility to add *composition filters* [8] at runtime. Composition filters can be used to manipulate or replicate messages without making the agents code any more complex. For our simulation we find very useful to replicate all the messages which add edges or nodes to monitoring agents which hold the model of the whole network and can compute the metrics or provide visualisation. With composition filters, the logic is not embedded with the simulation code and resides externally: this way it is more easily customisable.

7 CONCLUSIONS AND FUTURE WORK

In this paper, we have compared analytic metrics of different network generation models with data from real online social networks. We have found that some models account for some properties of the real OSNs. There is strong need for more analysis of real OSNs, possibly obtaining full data-sets, as crawls often have strong and difficult to evaluate biases.

The simulation system we have built, although at an early stage, is able to generate networks according to some basic models, but it is designed for extensibility. We plan to extensively test navigability on the networks we generate, potentially super-imposing some meta-model which guarantees that property over non-regular structures. We also plan to further explore behavioural generation models, exploiting agent-based models to reproduce patterns related to human behaviour.

Of course, it is still an open question the extent to which predictions valid for artificially generated social networks are valid for real online social networks. This issue is however not easily dealt with in general. However, results look promising [27] and with more sophisticated network models it is likely that simulation is going to be an increasingly important technique for social network related studies.

Moreover, our long term goal is using the generated networks to perform simulations. In this scenario, even though none of the models perfectly reproduces a real social network, performing simulations on many different synthetic networks generated with multiple models may lead to meaningful conclusions nonetheless.

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Time-Varying Graphs and Social Network Analysis: Temporal Indicators and Metrics

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Abstract. Most instruments - formalisms, concepts, and metrics - for social networks analysis fail to capture their dynamics. Typical systems exhibit different scales of dynamics, ranging from the fine-grain dynamics of interactions (which recently led researchers to consider *temporal* versions of *distance*, *connectivity*, and related indicators), to the evolution of network properties over longer periods of time. This paper proposes a general formal approach to study networks' structural evolution for both *atemporal* and *temporal* indicators, based respectively on sequences of static graphs and sequences of time-varying graphs that cover successive time-windows. All the concepts and indicators, some of which are new, are expressed using a *time-varying graph* formalism recently proposed in [10]. Experimental results of the application of atemporal metrics applied to a portion of the scientific community of arXiv are provided.

1 Introduction

Social networks have drawn a lot of attention in the past few years, and the analysis of their dynamics represents a pressing scientific challenge. The research efforts in this area strive to understand the driving forces behind the evolution of social networks and their articulations within social dynamics, e.g., opinion dynamics, the epidemic or innovation diffusion, the teams formation and so on ([7, 11, 14, 18, 27, 29, 33, 34, 35, 37, 38]). In other words, it is known that individuals are influenced (*e.g.* concerning their opinion) through their social network, it is also known that individuals take into account others' attributes when deciding to evolve their social network, but yet qualitatively not much is known concerning the dynamical patterns that are produced by such an interplay.

Curiously enough, everybody agrees on the stance that social networks are dynamic, e.g. individuals join, participate, attract, compete, cooperate, disappear, and affect the shape and strength of the network and its relationships. Yet, the current instruments (definitions, models, metrics) are mainly drawn for static networks and generally fail to capture the evolution of phenomena and their dynamical properties – *temporal dimension* – focusing instead on structural [23] or statistical aspects [39] of the systems. As stated in [28], the central problem in this area is the definition of mathematical models able to capture and to reproduce properties observed on the real networks.

The increasing availability of real datasets (*e.g.* e-mails logs, online forums, or meta-data on scientific publishing), as well as development of smartphones, vehicular networks, and satellite networks have recently fostered research on dynamic networks and caused the appearance of new dedicated concepts. In particular, early works around transportation and *delay-tolerant* networks (those networks characterized by an absence of instant end-to-end connectivity) have led to the concept of *journey* [6] - also called *schedule-conforming path* [2], *time-respecting path* [20, 24], or *temporal path* [12, 42, 43]. Journeys can be seen as a particular kind of path whose edges do not necessarily follow one another instantly, but instead induces waiting times at intermediate nodes.

A direct consequence of considering *journeys* instead of *paths* is that all the concepts usually built on top of paths can in turn take a temporal meaning. This includes the concept of *temporal distance* [6] - also called *reachability time* [20], *information latency* [25], or *temporal proximity* [26] -, which accounts for the minimal speed of information propagation between two nodes, and the concept of *temporal connectivity* [3] based on the existence of journeys. On the social network side, recent studies focused on measuring the temporal distance between individuals based on e-mail datasets [25, 26] or inter-meeting times [43]. Very recently, *temporal betweenness* and *temporal closeness* were also considered in a social network context in [32, 41]. All these *temporal* indicators complete the set of *atemporal* indicators usually considered in social network analysis, such as (the usual versions of) distance and diameter, density, clustering coefficient, or modularity, to name a few. It is important to keep in mind that these indicators, whether temporal or atemporal, essentially accounts for network properties at a reasonably short time-scale (*fine-grain* dynamics). They do not reflect how these properties evolve over longer periods of time (*coarse-grain* dynamics).

In this paper, we propose a general approach to look at the evolution of both atemporal and temporal indicators. Looking at the evolution of atemporal indicators can be done by representing the evolution of the network as a sequence of *static* graphs, each of which represents the aggregated interactions over a given time-window. Atemporal indicators can then be normally measured on these graphs and their evolution studied over time. The case of temporal indicators is more complex because the corresponding evaluation cannot be done on static graphs. The proposed solution is therefore to look at the evolution of temporal indicators through a *sequence* of shorter *time-varying graphs*, which are *temporal subgraphs* of the original time-varying graph, covering successive time-windows. We discuss several examples of indicators, both temporal and atemporal, some of which are new. The evolution of some atemporal indicators is accompanied with recent experimental results from [36], based on online data on scientific networking consisting of dated co-authoring and citation records. We first present the time-varying graph (TVG) formalism from [10], which we use to express all temporal concepts and evolution properties in a concise and elegant way. We then discuss the two suggested approaches to study the evolution of *atempo-*

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ral and *temporal* indicators, respectively.

2 Dynamic Networks as Time-Varying Graphs

This section presents the *time-varying graph* formalism (TVG) recently introduced in [10]. This formalism is semantically equivalent to other graph formalisms, like that of *evolving graphs* [16], but suggests in comparison an *interaction-centric* point of view. This point of view was also present in the time-labelling function of [24], but only for punctual contacts and latencies. The TVG formalism allows us a concise and elegant formulation of temporal concepts and properties.

2.1 The TVG Formalism

Consider a set of entities V (or *nodes*), a set of relations E between these entities (*edges*), and an alphabet L accounting for any property such a relation could have (*labels*); that is, $E \subseteq V \times V \times L$. The definition of L is domain-specific, and therefore left open –a label could represent for instance a particular type of relation in a social network, a type of carrier in a transportation networks, or a communication medium in communication networks. For generality, L is assumed to possibly contain multi-valued elements (e.g. *<satellite link; bandwidth of 4 MHz; encryption available;...>*). The set E enables multiple relations between a given pair of entities, as long as these relations have different properties, that is, for any $e_1 = (x_1, y_1, \lambda_1) \in E, e_2 = (x_2, y_2, \lambda_2) \in E, (x_1 = x_2 \wedge y_1 = y_2 \wedge \lambda_1 = \lambda_2) \implies e_1 = e_2$.

The relations between entities are assumed to take place over a time span $\mathcal{T} \subseteq \mathbb{T}$ called the *lifetime* of the system. The temporal domain $\mathbb{T}$ is generally assumed to be $\mathbb{N}$ for discrete-time systems or $\mathbb{R}$ for continuous-time systems. We denote by time-varying graph the structure $\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$, where $\rho : E \times \mathcal{T} \to \{0, 1\}$, called *presence function*, indicates whether a given edge is present at a given time, and $\zeta : E \times \mathcal{T} \to \mathbb{T}$, called *latency function*, indicates the time it takes to cross a given edge if starting at a given date.

Such a formalism can arguably describe a multitude of different scenarios, including:

- Transportation networks - e.g. aviation, where nodes are the cities, directed edges are regular flights, whose departure dates are given by *punctual* presences, and flight duration by non-nil latencies.
- Communication networks - e.g. wireless mobile networks, where an edge is present whenever its two endpoints are within range, the latency corresponding here to the time to propagate a message.
- Complex systems, among which social networks - e.g. scientific networks, where the nodes are scientists, and the edges (possibly both directed and undirected) account for example for citations or collaborations.

These examples illustrate the spectrum of models over which the TVG formalism can stretch. As observed, some contexts are intrisically simpler than others and call for restrictions (e.g. directed vs. undirected edges, single vs. multiple edges, punctual vs. lasting relations). Further restrictions may apply. For example the latency function could be decided constant over time, over the edges, over both, or simply ignored. In fact, a vast majority of work in social networks does not require such information (e.g., the propagation time of an email is of little interest to the understanding of a community behavior). Since the scope of this paper is social network analysis, we will deliberately omit the latency function and consider TVGs described as $\mathcal{G} = (V, E, \mathcal{T}, \rho)$.

2.2 Journeys and related Temporal Concepts

A crucial concept in time-varying graphs is that of *journey* which is the temporal extension of the notion of path, and forms the basis of most recently introduced temporal concepts. A sequence of couples $\mathcal{J} = \{(e_1, t_1), (e_2, t_2) \ldots, (e_k, t_k)\}$, such that $\{e_1, e_2, ..., e_k\}$ is a walk in G, is a *journey* in $\mathcal{G}$ if and only if $\forall i, 1 \le i < k$, $\rho(e_i, t_i) = 1$ and $t_{i+1} \ge t_i$. We denote by $departure(\mathcal{J})$, and $arrival(\mathcal{J})$, the starting date t_1 and the last date t_k of a journey $\mathcal{J}$, respectively. Journeys can be thought of as *paths over time* from a source to a destination and therefore have both a *topological* and a *temporal* length. The *topological length* of $\mathcal{J}$ is the number $|\mathcal{J}| = k$ of couples in $\mathcal{J}$ (i.e., the number of *hops*); its *temporal length* is its end-to-end duration: $||\mathcal{J}|| = arrival(\mathcal{J}) - departure(\mathcal{J})$.

Let us denote by $\mathcal{J}^*$ the set of all possible journeys in a time-varying graph $\mathcal{G}$, and by $\mathcal{J}^*(u, v) \subseteq \mathcal{J}^*$ those journeys starting at node u and ending at node v. In a time-varying graph, there are three natural distinct measures of *distance*, and thus three different types of "minimal" journeys.

- The *shortest distance* from a node u to a node v at time t is simply $d^t(u, v) = Min\{|\mathcal{J}| : \mathcal{J} \in \mathcal{J}^*(u, v) \wedge departure(\mathcal{J}) \ge t\}$.
- The *foremost distance* from u to v at time t is $\delta^t(u, v) = Min\{arrival(\mathcal{J}) - t : \mathcal{J} \in \mathcal{J}^*(u, v) \wedge departure(\mathcal{J}) \ge t\}$.
- The *fastest distance* from u to v at time t is defined as $\hat{\delta}^t(u, v) = Min\{||\mathcal{J}|| : \mathcal{J} \in \mathcal{J}^*(u, v) \wedge departure(\mathcal{J}) \ge t\}$.

A journey $\mathcal{J} \in \mathcal{J}^*(u, v)$ with $departure(\mathcal{J}) \ge t$ is said to be *shortest* at time t if $|\mathcal{J}| = \delta^t(u, v)$; *foremost* at time t if $arrival(\mathcal{J}) - t = \delta^t(u, v)$; and *fastest* at time t if $||\mathcal{J}|| = \hat{\delta}^t(u, v)$.

Whether in the contexts of social networks or communication networks, a number of higher concepts have been recently defined on top of these. They include new meanings of *connectivity* and *connected components* [3], *temporal eccentricity* and *temporal diameter* [6], or *temporal betweenness* and *temporal closeness* [41], among others. As discussed in the introduction, these concepts allow for novel insights on the way nodes interact at a small time-scale (*fine-grained* dynamics), but do not reflect the way the network *evolves* at over longer periods of time (*coarse-grain* dynamics).

3 Capturing the Evolution

In this section we introduce a framework to study the behavior of network parameters (or indicators) during the lifetime of a time-varying graph. Two types of indicators are described: *atemporal* and *temporal* ones. Atemporal parameters are defined on static networks and their evolution over time can be observed by measuring them over sequences of static graphs, where each graph of the sequence corresponds to the aggregation of interactions that occur in a given interval of time (we call them *footprints* of a TVG). Temporal indicators, on the other hand, are only defined on time-varying graphs, taking into account their temporal nature. The evolution of such indicators requires to consider a sequence of (non-aggregated) time-varying graphs, each of which corresponds to a *temporal subgraph* of the original one for the considered interval.

3.1 Evolution of Atemporal Indicators

3.1.1 *Methodological approach*

TVGs as a sequence of footprints. Given a TVG $\mathcal{G} = (V, E, \mathcal{T}, \rho)$, one can define the *footprint* of this graph from t_1 to

t_2 as the static graph $G^{[t_1,t_2)} = (V, E^{[t_1,t_2)})$ such that $\forall e \in E, e \in E^{[t_1,t_2)} \iff \exists t \in [t_1, t_2), \rho(e,t) = 1$. In other words, the footprint aggregates interactions over a given time window into static graphs. Let the lifetime $\mathcal{T}$ of the time-varying graph be partitioned in consecutive sub-intervals $\tau = [t_0, t_1), [t_1, t_2) \ldots [t_i, t_{i+1}), \ldots$; where each $[t_k, t_{k+1})$ can be noted τ_k. We call *sequence of footprints* of $\mathcal{G}$ according to τ the sequence $SF(\tau) = G^{\tau_0}, G^{\tau_1}, \ldots$. Considering this sequence with a sufficient size of the intervals allows to overcome the strong fluctuations of fine-grain interactions, and focus instead on more general trends of evolution. Note that the same approach could be considered with a sequence of intervals that are *overlapping* (*i.e.,* a sliding time-window) instead of disjoint ones. Another axis of variation can be considered whether or not the set of nodes in each G^{τ_i} is also varying, e.g. being restricted to nodes that have at least one adjacent edge in $E^{\tau i}$ (which is the case in the experimental results shown below).

Looking at atemporal parameters. Since every graph in SF is static, any classical network parameter (degree, neighborhood, density, diameter, modularity, *etc.*) can be directly measured on it. When observing the evolution of a parameter over SF, one can achieve different levels of granularity by varying the size of the footprint intervals. Depending on the parameter and on the application, different choices of granularity are more appropriate to capture a meaningful behavior. At one extreme, each interval could correspond to the smallest time unit (in discrete-time systems), or to the time between any two consecutive modification of the graph. In these cases every footprint corresponds to an instant *snapshot* of the network, and the whole sequence becomes equivalent to the *evolving graph* model [16]. At the other side of the spectrum, i.e. taking $\tau = \mathcal{T}$, the sequence would consist of a single footprint aggregating all interactions over the network lifetime.

3.1.2 Indicators and Discussions

We now discuss the definitions and peculiarities of a set of atemporal parameters, some of which are illustrated upon recent experimentations results (from [36]) on the hep-th (High Energy Physics Theory) portion of the arXiv website. The dataset consists of a collection of papers and their related citations over the period from January 1992 to May 2003. For each paper the set of authors, the dates of on-line deposit, and the references to other papers are provided. There are 352 807 citations within the total amount of 29 555 papers written by 59 439 authors. From the dataset we extract the network of the most proficient authors - i.e., the authors of papers which received more than 150 citations. In all the example charts, a one-year time window is used.

Evolution of the Density. One important and widely used indicator aimed at measuring the network structure is the density, which measures how close it is to a complete graph. The *density* of a graph $G = (V, E)$ is defined as:

$$D = \frac{|E|}{|V| * (|V| - 1)}$$

The *evolution of the density* could be observed by looking at its trend over the sequence of footprints $SF = G^{\tau_1}, G^{\tau_2}, \ldots, G^{\tau_i}$. The trend of this value reflects the network's topology formation during time from a global perspective. It could be useful in many cases, such as in the study of transportation networks, e.g. to see how the equipment (number of roads, railways, flights connections...) increases over time. Figure 1 provides another example showing a trend of *un-densification* observed in the above-mentioned scientific publishing network.

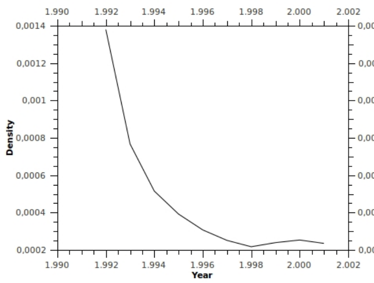

Figure 1. Evolution of the density.

This counter-intuitive trend can be explained by an increasing number of authors. (Recall that these experimentations considered that the set of nodes in the footprint sequence was varying among the G^{τ_i}s, based on the existence of adjacent edges in the considered footprints.)

Evolution of the Clustering Coefficient. The clustering coefficient is used in social network analysis to characterize architectural aspects. Several studies (e.g., [19, 44]) suggest that in general nodes tend to create tightly compact groups characterized by a relatively high density of ties. Roughly speaking, the *clustering coefficient* of a node indicates how close to a clique its neighborhood is. It is formally defined in [44] as

$$C(x) = \frac{|\{(u,v) : u, v \in N(x)\}|}{deg(x)(deg(x) - 1)}$$

The average clustering coefficient of a graph can then be defined as the average over all nodes:

$$AC = \frac{1}{|V|} \sum_{x \in V} C(x)$$

As for the density, the evolution of these properties could be observed through measuring it on the footprints of SF. An increasing or decreasing trend of clustering coefficient would typically capture the formation or dismemberment of social communities at a global scale. An example is provided on Figure 2, still with the same dataset, which shows that the connectivity first tends to be sparse, then after a phase transition around 1999, the nodes start to cluster into denser sub-communities.

Evolution of the Modularity. Modularity measures how the structure of a given network is modular, *i.e.,* how it can be decomposed into subparts. It also quantifies the quality of a given network division into modules or communities. Networks with high values of modularity are characterized by dense *intra*-module connections and sparse *inter*-module connections.

The *modularity* of a pair of nodes u and v is defined as

$$M(u,v) = \frac{deg(u) * deg(v)}{2|E|}$$

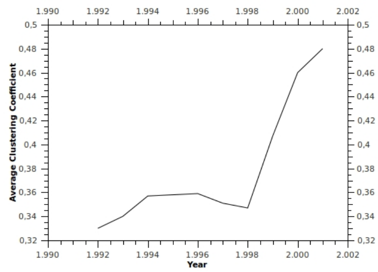

Figure 2. Average Clustering Coefficient Evolution

The most common use of modularity is the detection of community structures (e.g. [4]). Such an indicator, if observed over time, can provide very interesting hints for the analysis of complex dynamic networks, in particular for the evolution of their structures and groups formation. It could also enable to see whether communities tend to specialize and/or homogenize. Figure 3 shows the evolution of the *average* modularity over the sequence of footprints of our scientific networking example.

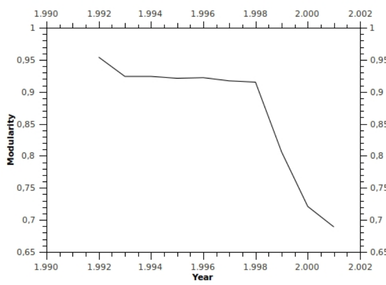

Figure 3. Evolution of the Modularity

In a similar way as for the clustering coefficient, the evolution of modularity exhibits a phase transition around 1999 that separates a monotone trend from a decreasing one. This means that nodes first tends to form separate groups, which at some point start to interconnect with each other into a smaller number of larger groups (formation of communities). Modularity and clustering coefficient are clearly related. It was shown for example in [1] that networks with the largest possible average clustering coefficient are found to have a *modular* structure, and at the same time, to have the smallest possible average distance between nodes.

Evolution of the Degree Power Law. Real world networks are "scale-free", in the sense that their node degree distributions follow a power-law that is not affected by the size of the network. Such a power law indicates that the fraction F of nodes that have degree k decreases as $F(k) \sim k^{-\gamma}$, where $\gamma \in \mathbb{R}$ is a parameter that varies among different types of networks; its value is generally in the interval $[2, 3]$.

The evolution of the power law over time could reflect for example the arrival or departure of hubs - nodes that interconnect several groups. Figure 4 shows the evolution of the power law exponent over the sequence of footprints of our dataset. As our example deals with the network of most proficient authors, i.e. a subset of the dataset, the values in Fig 4 are slightly different from the traditional reference values. In particular, the graphic shows how closely the degree distribution of a graph follows a power-law scale at each time interval. The higher the values, the more unequal is the distribution of connections within the nodes of the network.

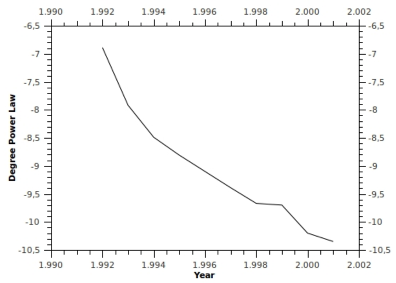

Figure 4. Evolution of the degree power law

Notice that the curve in Figure 4 provides additional details about the interaction pattern evolution of the network. As the evolution of the clustering coefficient shows an increase of the clustered structure of the network, and the modularity indicates that such an increase is characterized by the connection among separated groups, the decrease of the degree power law shows that the interconnection process is driven by nodes with low degree acting as hubs within groups.

Evolution of the Conductance. Social networks are intensively studied not only with respect to their structure, but also regarding the interactions occurring on top of them. For instance, several studies focused on *information diffusion* within groups based on a process of social influence (*influential networks* [21]). Such a process was intensively studied under the name of *viral marketing* (see for instance [15]) to predict the propagation time of a message over a network. It was recently shown in [13] that the *conductance* - a measure that characterizes the time of convergence of a random walk toward its uniform distribution - plays an important role in "push-pull" based dissemination strategies. The conductance of a graph is defined as the minimum conductance over all the possible cuts $(S, \bar{S})$ in this graph (a *cut* is a partition of the nodes into two disjoint subsets). The conductance of a cut $(S, \bar{S})$ is defined as

$$\varphi(S) = \frac{|(x_{\in S}, y_{\in \bar{S}}) \in E|}{\min(|(x_{\in S}, y_{\in V}) \in E|, |(x_{\in \bar{S}}, y_{\in V}) \in E|)}$$

The evolution of conductance might reveal how the links of a network are organizing according to the distance between nodes, and indirectly reflect a process of self-optimization (or deterioration) of the network efficiency.

3.2 Evolution of Temporal Indicators

3.2.1 Methodological approach

Most temporal concepts – including all those mentioned at the end of Section 2.2 – are based on replacing the notion of *path* by that of *journey*. As a result, they can be declined into three versions depending on the type of distance considered (*i.e.,* shortest, foremost, fastest). Since journeys are paths over *time*, the evolution of parameters based on journeys *cannot* be studied using a sequence of aggregated static graphs. For example, there might be a path between x and y in all footprints, and yet possibly no journey between them depending on the precise chronology of interaction. To analyze the evolution of such parameters, we need to use a more powerful tool: a sequence of time-varying graphs.

TVGs as a sequence of (shorter) TVGs. Subgraphs of a time-varying graph $\mathcal{G} = (V, E, \mathcal{T}, \rho)$ can be defined in a classical manner, by restricting the set of vertices or edges of $\mathcal{G}$. More interesting is the possibility to define a *temporal subgraph* by restricting the lifetime $\mathcal{T}$ of $\mathcal{G}$, leading to the graph $\mathcal{G}' = (V, E', \mathcal{T}', \rho')$ such that

- $\mathcal{T}' \subseteq \mathcal{T}$
- $E' = \{e \in E : \exists t \in \mathcal{T}' : \rho(e, t) = 1\}$
- $\rho' : E' \times \mathcal{T}' \to \{0, 1\}$ where $\rho'(e, t) = \rho(e, t)$

In the same way as for the sequence of footprints SF, we can now look at the evolution of a TVG through a sequence of shorter TVGs *ST*$(\tau) = \mathcal{G}^{\tau_0}, \mathcal{G}^{\tau_1}, \ldots$, in which the intervals are either disjoint or overlapping.

3.2.2 Indicators and Discussions

Evolution of the (temporal) Distance. The basic concept of this class of indicators is that of *distance*. In particular, there are three different types of distances - shortest, fastest, and foremost - that are respectively noted $d(u, v)$, $\delta(u, v)$, and $\hat{\delta}(u, v)$. As discussed in the introduction, these concepts of distance are central in various contexts and were recently subject to several studies. Algorithms to compute optimal journeys according to the three types of distances are available in [6]. (*Distributed* analogues of these algorithms were recently proposed in [8] and [9].) Computing the distance gives an idea of how reachable the nodes are from each other, and thereby constitutes a general bound on dissemination speed.

A concept symmetric to the one of temporal distance is that of *temporal view*, introduced in [25] in the context of social network analysis. The temporal view (or simply *view*) that a node v has of another node u at time t, denoted $\phi_{v,t}(u)$, is defined as the latest (i.e., largest) $t' \leq t$ at which a message received by time t at v could have been emitted at u; that is, in the TVG formalism,

$$\max\{departure(\mathcal{J}) : \mathcal{J} \in \mathcal{J}^*(u, v) \wedge arrival(\mathcal{J}) \leq t\}.$$

This concept could, as that of distance, be declined into three versions (the above one is symmetric to the *foremost* distance). Studying the evolution of temporal distances or views over a sequence of temporal subgraphs reflects how close in time, or in hops, the nodes tends to become. It serves as a basis to most of the indicators discussed below.

Evolution of the (temporal) Diameter and Eccentricity. The three journey-based versions of eccentricity and diameter were first discussed in a communication network context [6]. The eccentricity of a node u in a TVG $\mathcal{G}$ can be defined in terms of *shortest* journeys as

$$e(u) = \max\{d(u, v) : v \in V\}$$

where d can be substituted by $\delta(u, v)$ or $\hat{\delta}(u, v)$ to obtain the foremost eccentricity $\varepsilon(u)$, or the fastest eccentricity $\hat{\varepsilon}(u)$, respectively. The eccentricity of a node directly reflects its reachability capacity, and therefore the impact it can have on the network. Such a parameter could have a particular significance in some field of research, e.g. in epidemics, the existence of nodes with a high temporal eccentricity could be associated with the possibility for a virus to survive long-enough to reinfect people. Three versions of the diameter naturally follow based on those of eccentricities: $max\{e_i(u) : u \in V\}$, $max\{\varepsilon_i(u) : u \in V\}$, and $max\{\hat{\varepsilon}_i(u) : u \in V\}$. The foremost version of the temporal diameter was specifically studied in [12] from a stochastic point of view by Chaintreau *et al.*, but to the best of our knowledge, the *evolution* of the temporal diameter or eccentricities have never been considered yet. Looking at them could reveal complex social parameters, e.g., considering the evolution of standard deviations among node eccentricities could reflect how a network tends to create *fairness* or *inequalities* among its participants.

Evolution of the (temporal) Centrality. One of the most important properties of social networks' structures is the so-called notion of *power*. As a shared definition of power is still object of debate, the design of metrics able to characterize its causes and consequences is a pressing challenge. In particular the social network approach emphasizes the concept of power as inherently relational, *i.e.,* determined by the network topology. Hence, the focus must be put on the relative positions of nodes. In order to characterize such a property the concept of *centrality* has emerged. The simplest centrality metric, namely the *degree centrality*, measures the number of edges that connect a node to other nodes in a network. Over the years many more complex centrality metrics have been proposed and studied, including *Katz status score* [22], α-*centrality* [5], *betweenness centrality* [17], and several others based on random walk [30, 40], the most famous of which is the *eigenvector centrality* used by Google's PageRank algorithm [31]. The temporal adaptation of these concepts is meaningful, and Kleinberg et al. have shown in [25] that nodes that are topologically more central are not necessarily central from a temporal point of view, whence the concept of *temporal centrality*. Studying the evolution of these over time could in turn shed light on how "powerful" nodes tends to emerge in a network. *Betweenness* and *closeness* are two well-known measures of centralities.

Temporal betweenness. The betweenness of a node in a static graph measures the occurrences of that node within the shortest paths of other nodes [17]. A temporal version of the betweeness based on foremost journeys was considered in recent work by Tang *et al.* [41]. The definition can be generally formulated as

$$B(q) = \sum_{v \neq u \neq q \in V} \frac{|d'(u, v, q)|}{|d(u, v)|}$$

where $|d(u, v)|$ is the number of shortest journeys between u and v in the time varying graph $\mathcal{G}^{\tau_i}$, and $|d'(u, v, q)|$ is the number of shortest journeys, among them, that pass through q. We can analogously define the temporal betweeness in terms of foremost or fastest distance, by substituting $d(u, v)$ with $\delta(u, v)$ or $\hat{\delta}(u, v)$.

Temporal closeness. In a static context, the closeness measures the mean of the shortest paths between a node and all the other reachable nodes [17]. It can be formally defined as

$$TC(u) = \sum_{v \in V \setminus u} \frac{d(u, v)}{|\{w \in V : \exists \mathcal{J} \in \mathcal{J}^*(u, w)\}|}$$

and again, possibly declined to a shortest, foremost ($\delta(u, v)$), or fastest ($\hat{\delta}(u, v)$) versions. As one will certainly notice, this parameter is highly related to that of temporal eccentricity, and yet, both have appeared in very different fields of research. This illustrates again how general both the temporal concepts and the formal tools can be.

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Adaptive Security Scheme for Open Networks

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Abstract. Existing security solutions have had only limited success in addressing the diversity of attacks on different types of open and decentralised networks. Furthermore, they do not differentiate between intentional violation and unintentional or accidental malfunction. As an alternative to 'lock down', we propose a generic, re-configurable and adaptive network security scheme. This scheme combines social networks with multi-agent systems, by interleaving opinion formation and belief revision processes in an agent's architecture. The operation of the proposed scheme is animated with a possible application to a security problem in an open system.

1 INTRODUCTION

Open systems and networks offer substantial advantages in terms of scale, opportunity and generativity. They can be operationally successful because the system design is predicated on an assumption of co-operation, and as a result conflict resolution can be pushed to either the application layer or the physical layer [23]. The system is then tolerant of transience, mobility, resource contention, heterogeneity, and accidental malfunction, and can recover from sub-ideal behaviour [1].

The real problems start when the assumption of cooperation is void. These problems range from selfish behaviour due to conflicting goals, through to deliberate, malicious behaviour to disrupt or destroy the system. In addition, targeted attacks can be launched on a network. They come either from outside and usually compromise several agents, or from agents within the network who can even join their forces for a more effective attack. Possible attacks include denial of service, data tampering or resource depletion. Several ways to defend an attack are described in the literature, but most methods are tailored to either specific types of attacks or specific networks, as it is impossible to address all of them at once.

Arguably, existing security solutions have had only limited success in addressing the diversity of attacks on different types of open and decentralised networks. For example, computationally intensive key-based authentication schemes are inappropriate in resource-limited environments such as sensor networks. Similarly, insurance and protection schemes are predicated on knowing the nature of the attack beforehand, which is dependent on a pre-emptive event recognition mechanism. Furthermore, lacking a 'cognitive' dimension, they do not easily differentiate between intentional violation and unintentional or accidental malfunction. Finally, there is always the risk of doing the attackers job for them. For example, if the security response to a *potential* battery exhaustion denial-of-service attack in an ad hoc network itself consumes excessive resources, then the attacker does not even need to launch an attack to bring down the network. The system's *own* response to the threat alone is enough.

Therefore open system designers must anticipate both accidental misbehaviour but best efforts to restore functionality, and intentional misbehaviour yet best efforts to destroy functionality. As an alternative to 'lock down', i.e. to reject the open systems concept, a security scheme is required that:

- complements cryptographic and game-theoretic techniques,
- is generic and scalable to different classes of networks,
- differentiates between intentional and unintentional errors, and
- adapts to a changing environment.

In previous work [6], we proposed to interleave social networks and multi-agent systems based on norm-governed specifications. Opinion formation was used to determine regulations for agent behaviour from an *external* perspective and could prove useful in dealing with selfish behaviour which was non-compliant with system specifications.

In this work, we propose to extend the scheme for open and ad hoc networks. The new security scheme is intended to be applicable to different types of networks, and to allow for detection and recovery from concerted attacks as well as unintentional errors and/or selfish behaviour. To this end, complementary aspects of multi-agent systems and social networks are integrated into the *internal* architecture of an agent, to evaluate the behaviour of other agents according to the agreed regulations. In particular, agents exchange their limited perception of the environment, and by taking the views of their neighbours into account, they revise their beliefs about an appropriate security policy.

The rest of the paper is organised as follows. The second section introduces different types of open networks, classifies them and highlights their security issues with emphasis on intentional and unintentional errors. Section 3 presents an open network to point out the necessity of the proposed security scheme. The scheme itself is then discussed in section 4 and envisioned in section 5. Afterwards, conclusions and further work are mentioned in section 6.

2 SECURITY IN OPEN NETWORKS

The networks in question are open, decentralised and heterogeneous. That means agents are joining and leaving the network anytime and don't have a publicly known internal architecture. They might be selfish and pursue conflicting goals, or they might just be unable to perform specific tasks. This can be caused by accident, necessity or design, but all aspects lead to a sub-ideal behaviour of the network.

2.1 Types of open networks

We identify three different types of open networks.

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A *wireless sensor network* (WSN) is formed by a collection of sensor nodes that are distributed in the environment and are typically resource constraint. They sense events like temperature, particle density or speed, and process this information via the (ad hoc) network (see [4]). The nodes then either perform actions autonomously, like the intervention in a production process, or collect and forward data to a base station. How information is processed in particular depends on the networks' specifications. There can be a determined or perpetually redefined node hierarchy, sensors can be fixed or mobile, alert or sleeping, etc. Summarised, the topology in the underlying graph can be ever-changing and algorithms have to be tailored accordingly.

Vehicular ad hoc networks (VANet) provide a service for traffic management, passenger safety and driver assistance. Sensors are installed in vehicles to monitor unexpected changes in the environment. These might be an accident behind a curve, a traffic jam or a dangerously overtaking car. To process the individually sensed information, vehicles in the vicinity form an ad hoc network and then communicate with either other vehicle clouds or fixed road side units. The main difference to most WSNs is the sporadic connectivity and the relatively short contact time between vehicles. [16] provides important details about the nature of and security problems in VANets.

A *Virtual Organisation* (VO) is a way to manage projects and companies in a decentralised manner. Collaborating parties can be from multiple disciplines to share their knowledge and access to facilities, such as data bases, software and computing power (see i.e. [7]). The main advantage is that employees can work from any location at any time. Furthermore a VO can quickly be set up to the needs of specific tasks across the boundaries of various physical or virtual institutions. Typically, a VO is organised as decentralised as possible in order to maintain scalability and mobility since projects involve more and more people and span over the whole planet. To run a company under such conditions VOs base very much upon trust and reputation, whereas the notion of trust occurs in different layers. Not only in computing devices, but also on an interpersonal level, therefore different schemes to verify authentication and integrity are needed.

2.2 Network classification

Table 1 compares important features that help characterising WSNs, VANets and VOs. These features represent the most important aspects, but can be further extended. Depending on the exact specifications of the considered network, the given values may vary as well.

The table demonstrates different classes of applications for which open systems and networks are appropriate and advantageous. However, it also demonstrates the wide variance in specific properties, and for many dependent properties, the extent to which these properties are contingent on the application or operating environment.

The range and scale of this diversity raise specific issues in dealing with intentional security attacks, as considered in the next section.

2.3 Security issues

As with any other network, open networks face potential security attacks. We expect an open network to have the following security objectives (i.e. [18]): Data Confidentiality, Data Authentication, Data Integrity, Data Freshness or Availability and Graceful Degradation. That means the targets are manifold. There is the node itself, the communication stack, traffic or service, key management protocols, identities, synchronisation protocols, etc. Depending on the specific characteristics they become even more vulnerable, for example due to limited resources or lack of authentication.

Many of the attacks are targeted on the communication stack: jamming of the physical layer, targeted attacks on the protocol, and flooding or desynchronisation on the transport layer (i.e. [22]). Furthermore the network layer is vulnerable to wormholes and spoofed, altered or replayed packets, as well as selective forwarding. To achieve a denial of service, an attacker for example floods the network with packets or sends a huge amount to a specific target in order to decrease the performance. This might in return lead to severe damages in the real world, depending on the purpose of the network. The list of possible attacks is by far not complete. Furthermore, depending on the type of network, more specific attacks come into play.

Another problem of networks where any node can join is that some devices are simply unable to meet the system requirements all the time. Thus, *unintentional errors* might occur. However, they might also occur due to security attacks. Therefore we have to deal with unintentional errors caused by accident or necessity, unintentional errors induced by intentional security attacks, and intentional errors.

2.4 Typical security approaches

In closed or centralised networks, error detection proves to be a valid approach to fight against intentional and unintentional errors. The outcome can then be used to modify the networks' regulations and security settings. In open, decentralised and autonomously working networks, error detection is no longer useful in most of the cases. Different security approaches have been investigated so far. Among the defences range prevention via protection or damage limitation via insurance, i.e. against epidemics or specific targets and links. Numerous papers look into their game theoretic aspects, such as [3, 8, 19]. The major drawback of these mechanisms is, that the type of attack has to be known beforehand to efficiently secure the network. Moreover unintentional failures are not taken into account, which can lead to perturbations of the systems that destabilise the Nash equilibria.

A standard in network security is cryptography. Most schemes rely on an existing shared secret basis of two agents in order to exchange keys for securing their actual network traffic. This is a problem when it comes to mobile or vehicular ad-hoc networks, as nodes do not have any prior contact information when they first access the network or move around and discover "new" nodes.

Khalili *et al.* [11] propose a scheme for ad-hoc networks that doesn't rely on any pre-shared keys and reduces the amount of messages that have to be sent in order to exchange a key. The keying mechanism uses network coding, which allows every agent to reconstruct the key by recombining information obtained from a fixed number of nodes. This is a powerful tool for closed systems, where an adversary has to compromise more nodes than the fixed amount, but in open networks this information is accessible for everybody.

Other approaches include countermeasures like package leashes, client puzzles, authentication or encryption schemes, see [18]. But as already indicated above, none of the investigated methods to secure ad hoc networks suited our needs. The main constraints therefore are the openness, heterogeneity and unpredictability of open networks. Moreover current security schemes do not or cannot distinguish intentional and systematic violation from unintentional failures, and are often tailored to a specific network.

2.5 Summary

Summing up, there are several different open networks and many different attacks that can be launched at these networks. Although most of the network/attack combinations have an existing security

Table 1. Comparison of selected network characteristics

Network feature	Sensor Network	VANet	Virtual Organisation
resource constrained	yes	no	a few devices
time constrained	usually yes	definitely	depends on business
mobility	limited	high, but organised	yes, but slow
data authentication	preferable	required	required
sender authentication	required	difficult (privacy)	required
hops	multi	multi + single	single
base stations	optional	yes	n.a.
communication range	short	short + long	variable
destination of information is specific node	depends	no	often yes
destination of information is physical area	depends	yes	usually not
error tolerance	depends	none	depends
key distribution	at initialisation	challenging	by trusted parties
user density	depends on app	low to start with	n.a.
human interaction	yes	better not	high

solution, it is almost impossible to anticipate what type of attack is going to happen in decentralised networks. That makes many approaches, like insurance for example, inapplicable and existing security schemes only offer a partial solution. Furthermore they do not have sufficient flexibility to distinguish between intentional and unintentional errors.

Draief *et al.* [6] present a scheme where a social network is combined with a norm-governed system. Agents correlate in different roles to self organise their network by agreeing on external rules, so to pursue personal and network goals. We propose to extend this scheme by considering the *internal architecture*, based on the same idea of agents assessing each other's behaviour with respect to these agreed external rules. Thus a configurable and adaptive scheme is produced that can be tailored to different network types and responds to security threats at runtime.

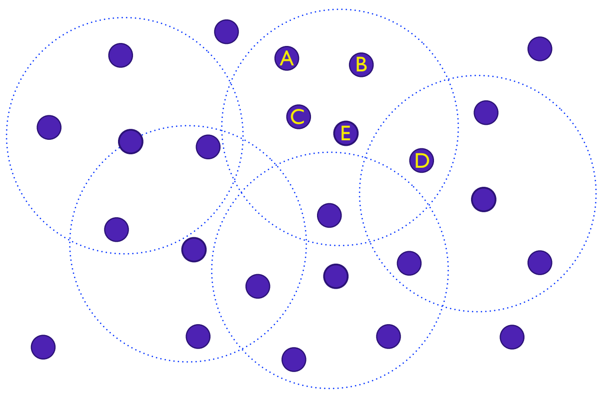

Figure 1. Working units in an open network

3 SCENARIO

This section introduces a simple open network where the security scheme will be applied to later on.

Imagine an open network, see figure 1, where the nodes, or working units, have to deliver packets to various destinations, but have only a limited communication range. Thus, the packets have to be sent via intermediate units before they reach their destination. If a unit forwards a packet successfully it gets a certain payoff, the same holds for an own packet that reaches the final destination $\mathcal{D}$.

The network can be represented as a multi-agent system

$$\mathcal{M}_t = \langle \mathcal{A}_t, ACT, \mathcal{N}_t \rangle \ ,$$

where $\mathcal{A}_t$ is the set of working units at time t, ACT is the set of possible actions and $\mathcal{N}_t$ is the adjacency list at time t, or rather the units in the neighbourhood that are able to reach each other.

Several actions that can follow upon events or past actions of the neighbourhood in each time slice are possible. These are $ACT = \{wait, forward, accept, drop\}$, and the main event is *packet gets ready for delivery* along with the status variables *stack*, for ready packets, and *payoff*. To ensure the packets to be forwarded, the following algorithm is executed.

In every time-slice t, a packet gets ready with a certain probability $\wp$ and a unit is in one of the three stages:

1. *Unit is idle (or waiting for packets to be forwarded to them)*
2. *Unit tries to forward, preferably to a unit nearest $\mathcal{D}$*
3. *Unit is waiting for their forwarded packet to be accepted*

The rules during each stage are:

1. `if` packets are forwarded to them, accept one:
   - if packet got here too often already: unit drops it $\xrightarrow{t+1}$ 1
   - otherwise $\xrightarrow{t+1}$ 2

   `if` a packet is remaining in stack $\xrightarrow{t+1}$ 2
   `else` $\xrightarrow{t+1}$ 1
2. `if` the destination is $\mathcal{D}$ $\xrightarrow{t+1}$ 1
   `if` target unit is not in stage 1:
   - if too many delivery attempts: unit drops the packet $\xrightarrow{t+1}$ 1
   - otherwise choose a different unit $\xrightarrow{t+1}$ 2

   `else` target unit is in stage 1 $\xrightarrow{t+1}$ 3
3. `if` packet is not accepted:
   - if too many delivery attempts: unit drops the packet $\xrightarrow{t+1}$ 1
   - otherwise choose a different unit $\xrightarrow{t+1}$ 2

   `else` packet gets accepted $\xrightarrow{t+1}$ 1

Incoming packets have priority over own packets but if there are packets from more than one unit coming in, some of the packets have to be refused. Those units then have to look for a different unit to forward their packet to. In case they tried every neighbour a certain amount of times, they are allowed to eventually drop the packet.

The algorithm can be tailored to any specific network by using the characteristics mentioned in table 1. As an example, the working units and packets can be replaced with the following arguments:

	WSN	VANet	VO
unit	sensor	car	service
packet	reading	TCP/IP-packet	service-request

The forwarding approach of the units is effective under the assumption of cooperation, not if a unit starts to act selfishly, for example floods the network with own packets (denial of service), drops every incoming packet, or spoofs them to get a higher payoff for "forwarded" packets. One can imagine various of the previously mentioned attacks to happen in the network. But not only intentional errors are possible, a working unit might just be short of resources and therefore not be able to forward the packets according to the protocol.

Thus it is essential for the units to communicate and to smartly use this information to revise their beliefs of the environment.

4 PROPOSED SECURITY SCHEME

4.1 Motivation

How can gossiping help to distinguish between different types of errors? Consider the network of figure 1, especially the units A to E. If unit A is experiencing *drop* by E at time t, that might be considered as an unintentional error, but if a drop happens at t, $t+1$, $t+2$ and $t+3$, it looks like intention. Drops can be detected by integrating a *drop-count* variable into the unit and prevented by punishment mechanisms. Now assume that E drops a packet of A in t, a packet of B in $t+1$, and so on. For each unit this looks like a one-off unintentional error and no countermeasures will be instantiated. But by gossiping about the unreliable unit E, units $A..D$ will eventually find out that E behaves intentionally maliciously.

This shows that communication is necessary for open systems to solve security problems, as well as revising and adopting epistemic beliefs according to the changing environment.

In order to address intentional and unintentional errors of the working units, the multi-agent system gets extended with a social network where everybody is able to gossip and reason about successful deliveries, unreliable neighbours or other issues that appear in the network. This helps the units to find out who the disrupting entities are and whether they are malicious or malfunctioning. It then allows them to take appropriate countermeasures, like excluding the malicious units from the network, but gives malfunctioning units another chance.

4.2 Adaptive network security scheme

The proposed scheme is as follows:

An open network, such as from the scenario in section 3, is extended with a social network where everybody is able to gossip and reason about successful forwarding, unreliable neighbours or other issues that appear in the network (see [6]). This enables them to find out who the disrupting entities are and allows them to take appropriate countermeasures, like excluding malicious nodes from the whole network whilst adjusting requirements for malfunctioning ones.

The three mechanisms behind gossiping and reasoning are:

- opinion formation
- belief revision including forgiveness
- action selection

Every agent A has a set of beliefs $\Delta_A(t)$ that contains views about the state of the network, other agents, principles, problems in question, actions, etc. From time to time, they want to find out whether their beliefs are still valid or should be updated, or whether new topics of discussion emerged in the neighbourhood or the changed state of the network augments the scope of actions. Every agent A also holds a set of opinions $\mathcal{O}_A(t)$ that he uses to form his own opinion depending on different opinions that other agents might have on a specific topic, moreover it influences the beliefs. Using the pool of different beliefs, an agent can then select the most appropriate action to achieve the pursued goals.

The new architecture of the multi-agent system will be denoted as

$$\mathcal{A}_t = \langle U, \Delta, \mathcal{O}; f_1, f_2, f_3 \rangle_t ,$$

where

$$\begin{aligned} U(t) &= \text{utility function} \\ \mathcal{O}(t) &= \text{opinion base} \\ \Delta(t) &= \text{belief database} \\ f_1(t) &= \text{opinion formation} \\ f_2(t) &= \text{belief revision} \\ f_3(t) &= \text{action selection} \end{aligned}$$

Figure 2 illustrates how agent A gets from an issue in question, here ψ, to specific actions.

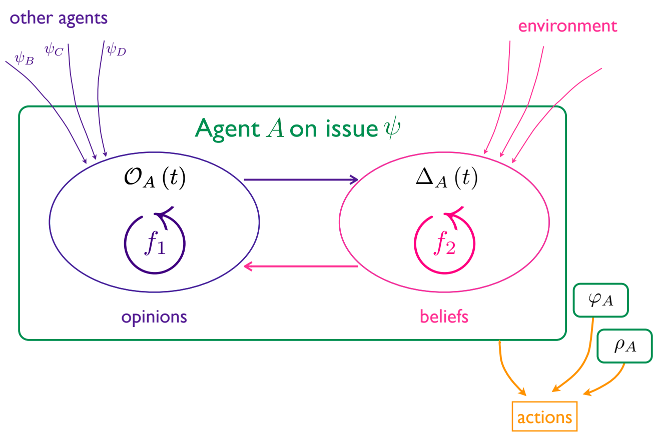

Figure 2. Internal architecture of an agent

The specific mechanisms that are chosen for f_1, f_2 and f_3 are left to the user.

Thus we obtain a generic, configurable and adaptive scheme to enhance security in open networks. Meaning, by adjusting the corresponding characteristics from table 1, the scheme can be used for any network, especially either of the mentioned types from section 2.1, and will then adapt to changes in the environment at runtime.

5 ENVISIONMENT

In this section, we perform a sort of 'thought experiment' to illustrate the interleaving of the opinion formation and belief revision modules and its application to a security problem in an open network.

5.1 Scenario

Consider five agents $A..E$ connected in an arbitrary network. Suppose E drops a packet from A. In a one-off encounter, A cannot be sure if this was an intentional or unintentional violation of the $forward\_packet$ rule. A gives E the benefit of the doubt. Suppose though E drops one packet each from A, B, C and D. In isolation, each of them gives E the benefit of the doubt. But: "to lose one looks like misfortune; to lose two looks like carelessness; to lose four looks like intention". If the four agents could pool their experiences, they might not be so forgiving.

Therefore, as a first pass of the proposal, we will instantiate process f_1 with a simple opinion formation model, and process f_2 with a forgiveness module which uses collective knowledge to revise subjective beliefs.

5.2 Opinion Formation

For example, process f_1 could be a variation of the Discrete Agent Model of Krause [9]. In this model, each agent $a \in \{A..E\}$ maintains a real value $a_i(t)$ which represents a's 'opinion' or 'position' on issue i at time t. $\widetilde{a}_i\,(t)$ and λ are linked to the belief revision part. ε is a bounding threshold, and there may be n issues, $1 \leqslant i \leqslant n$.

These values are synchronously updated in discrete time steps according to the equation:

$$a_i(t+1) = \lambda \cdot \frac{\sum_{b:\, d_t(a,b) \leqslant \varepsilon} b_i(t)}{\sum_{b:\, d_t(a,b) \leqslant \varepsilon} 1} + \widetilde{a}_i\,(t) \;\;, \quad b \in \{A..E\}\,,$$

where $d(.,.)$ measures the distance in a chosen (i.e. physical) norm.

That way each agent updates its opinion on issue i in the next time slice by computing the average value of its neighbours' opinions in the current time slice. This is done for each issue.

Note this model has been extended to a continuous agent model in [2] and in [15] to consider the 'trustworthiness' of the opinions' sources by considering the affinity to, and confidence in, one agent to another. However the above model is suitable for present purposes.

5.3 Forgiveness

Action selection in an open network is a trust decision: it is a willingness to expose oneself to risk. To make such a decision, it is necessary firstly to hold two beliefs [10]: that there is a rule, and that someone else's behaviour will conform to that rule; and secondly to make a computation: what is the probability that someone's behaviour will conform to that rule, and what is the benefit/cost if someone's behaviour does/does not conform to that rule [13].

For this to be a trust decision there has to be an element of risk: if the error in the trust decision is zero, it is not a trust decision. Therefore, there is always a possibility that the decision may be wrong, and an essential element of trust, often overlooked, is what to do when the trust decision is wrong.

In [20, 21], a forgiveness mechanism was proposed for decision-making about violation of norms. This was not based on reputation, which is a quantitative punishment mechanism, but instead on forgiveness, which is a qualitative repair mechanism. From psychological literature, forgiveness is known to stimulate voluntary acts of recompense, reduce a negative predisposition towards an offender, and accentuate a positive motivation for self-repair.

The forgiveness framework defined in [21] comprises eleven constituent signals (severity, frequency and intent of the offence; apology or reparation; utility and frequency of beneficial relationship; and familiarity, similarity, and shame or embarrassment) underlying the four positive motivations relating to the nature of the offence, remedial action, historical record and empathic relationship. This was implemented using a fuzzy inference system (FIS) which used fuzzy rules to compute a fuzzy value for each of the four positive motivations from the respective signals, which were themselves combined by a FIS to output a forgiveness decision (see Fig. 3).

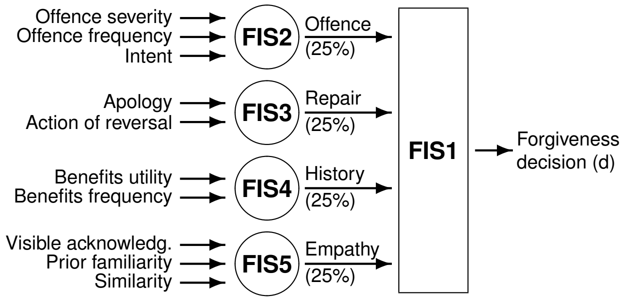

Figure 3. Forgiveness framework

5.4 Example

We can now see how these two processes can be interleaved. Suppose instead of the 11 constituent signals being related to subjective experience, they were also opened up as issues in the opinion formation model, i.e.:

$$\begin{aligned} A_1 &= \mathit{offence\_severity} \\ A_2 &= \mathit{offence\_frequency} \\ \ldots & \\ A_{11} &= \mathit{similarity} \end{aligned}$$

and likewise for agents B, C, etc.

Let us now suppose that the opinion of an agent on the frequency of an offence at time t is given by $\sum_{t_o} 1/2^{t-(t_o+1)}$, where t_o are the times of offences. Note at the time of evaluation t, opinion will be reported at time $t-1$, so the offence will have occurred at time $t-2$, so the opinion will by 1/2 if an offence occurs in the last but one time slice, 1/4 in the time slice before that, and so on; and so sums to 1 if there is an offence in *every* time slice. (There can only be one drop packet violation per time slice: if there is a packet in the agent's queue and there is no forward packet event.)

Now, for the first dropped packet, A might trigger the rule in FIS2:

if *severity* is low **and** *frequency* is low **and** *intent* is low
then *judgment of offence motivation* is 0.2

and because the frequency is 1/2 its value is considered low and a low value will be given to this motivation, therefore increasing the likelihood of forgiveness.

If A suffers no more dropped packets, then the frequency of offence will start low and rapidly tail off.

However, by aggregating the opinions using the equation above, by the time of the fourth dropped packet, and depending on how the fuzzy membership function for this signal for A has been defined, it might be now that A's opinion of the frequency (of offence) is high, and we might trigger the rule in FIS2 that:

if *severity* is low **and** *frequency* is high **and** *intent* is low
then *judgment of offence motivation* is 0.6

and consequently this could lead to a very different forgiveness decision from FIS1.

Possibly, this mechanism is prone to manipulation and we have to ensure that gossiping and forgiveness mechanisms cannot themselves be exploited for other types of attacks. However, no agent necessarily knows another's membership function so the number of packets that could be 'safely' dropped over any given time period cannot be computed in advance. In any case, the general security and error-handling principles can be seen to be at work: forgiveness for one-off or unintentional norm violations, no forgiveness for systematic of intentional norm violations.

6 SUMMARY & CONCLUSIONS

In summary, this is a position paper that proposes an extension to a social networks and multi-agent systems adaptive security framework, by exploiting the *internal architecture* of an agent. We considered how opinion formation, based on gossiping principles and algorithms from social networks, could be interleaved with belief revision algorithms based on multi-agent systems principles, such as autonomy (local control over local beliefs and decisions) and autonomicity (the proposed forgiveness framework is essentially a self-repair mechanism).

Note that we do not contradict the assumption that the internal structure is unknown in open systems. All the security scheme predicates is the communication of an opinion and of a decision; how these mechanisms are actually implemented is unknown. We have given one instantiation which (by animation) appears to help deal with intentional drop-packet actions in a common open network scenario.

At this point, this work is still ongoing, and we have at least four specific steps of future work to prove the concept contained in the proposal. The 'thought experiment' is not enough, therefore we need to implement the current proposal to test its properties, in MatLab or other agent/network simulation environments such as PreSage [12]. Then we will investigate other opinion formation models such that of [15] and its interleaving with epistemic belief revision algorithms specified by [5]. Furthermore, we will determine whether the parameters representing the networks' characteristics (see Table 1) make the scheme effective against different types of attacks in different types of networks. Finally, we need to deploy the mechanisms in real networks to their actual performance, as it is often the case that mechanisms such as we propose operate differently 'in the field' than in the lab. The pay-off from a successful investigation will be to disentangle one-off unintentional error from intentional malpractice, from repeated unintentional errors, and so on.

In normative and social systems, rules and regulations, and individual behaviour with respect to those rules and regulations, are open to interpretation, latitude, and license. For example, a fundamental principle of Robert's Rules of Order [17], the standard definition of keeping order in deliberative assemblies, meetings, etc., is that "anything goes unless someone objects". Furthermore, the forgiveness mechanism proposed here was inspired by a thorough study of the role of forgiveness in restoring order in social systems in the psychological literature.

We have tried to reproduce those mechanisms in our proposed adaptive security scheme for open networks. The alternative, as pointed out in [23], is that computer code itself becomes law, in which case various forms of perfect enforcement are available, for example by pre-emption, injunction, and surveillance. However, these security mechanisms, create a lock down and while supposedly eliminating 'bad' behaviour and preventing security attacks, also curtail good behaviour, such as generativity (a system's capacity to produce unexpected, unanticipated and un-designed-for change [23]). We see the same problem with open networks: security mechanisms aimed at 'perfect enforcement' using techniques such as key-based authentication or game theory might, in principle, eliminate 'bad' behaviour and mitigate security attacks. On the other hand, they will almost certainly curtail the potential advantages of open systems and networks, specifically those relating to organised adaptation and the emergence of complex behaviour [14].

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Rooting opinions in the minds: a cognitive model and a formal account of opinions and their dynamics

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Abstract. The study of opinions, their formation and change, is one of the defining topics addressed by social psychology, but in recent years other disciplines, like computer science and complexity, have tried to deal with this issue. Despite the flourishing of different models and theories in both fields, several key questions still remain unanswered. The understanding of how opinions change and the way they are affected by social influence are challenging issues requiring a thorough analysis of opinion per se but also of the way in which they travel between agents' minds and are modulated by these exchanges. To account for the two-faceted nature of opinions, which are mental entities undergoing complex social processes, we outline a preliminary model in which a cognitive theory of opinions is put forward and it is paired with a formal description of them and of their spreading among minds. Furthermore, investigating social influence also implies the necessity to account for the way in which people change their minds, as a consequence of interacting with other people, and the need to explain the higher or lower persistence of such changes.

1 Introduction

The studies about opinions, persuasion and social influence are foundational and pressing issues in social psychology; however, within this discipline, the dynamics of opinions at the level of population has been underestimated. There are also other disciplines that have shown a great interest regarding such an issue, ranging from political science ([17]) passing through socio-physics ([7]) up to complexity science ([18]). Understanding opinions, describing how they are generated and revised, and how fare opinions travel over the social space both as a consequence of social influence and as one of the main means through which social influence unfolds, is crucial for grasping a deeper understanding of human social cognition and behaviors.

Investigating opinions requires to take into account two levels of explanation: the individual and the social level. Social psychology has been mainly interested in explaining this first level, trying to describe the complex interplay of affective, cognitive and behavioral aspects that make opinions emerge. On the other hand, scholars from computer science and physics have tried to explain how different opinions can coexist or how they are modified through social interactions, treating opinions as objects that are exchanged and revised according to certain mechanisms that are quite far from the reality of cognitive and social processes. In both cases there is a reductionist fallacy that works in apparently different ways but it affects both these approaches, leading them to treat opinions either as a set of unrelated specific elements or as a unidimensional object that has nothing in common with a cognitive representation.

We claim that opinions are highly dynamical representations resulting from the interplay of different mental representations and affected by the mental states of other individuals in the same network. Aim of this work is to provide an interdisciplinary account to describe how social influence leads to opinion formation, evolution and change. Moving from a characterization of opinions as mental representations with specific features, we will try to model how opinions are generated within the agents' minds (micro-level) and how they spread within a network of agents (macro-level). When explaining the emergence of macro-social phenomena we need to know what happens at the micro-level, i.e. what drives human actions and decisions in order to understand how individuals' representations and behaviors can give rise to socially complex phenomena and how those affect agents' actions. Without explaining how opinions are formed and manipulated within the individuals' minds, it is very difficult to account for the way in which they change as an effect of social influence. Our aim is to understand whether and how heterogeneous agents, endowed with different beliefs and goals, may come to share a given viewpoint and what consequences this sharing has on agents' behaviors. We are interested in providing answers, at least partially, to the following questions: What is an opinion? What mechanisms lead people to change their opinions? How can individuals resist to changes? What are the mechanisms of influence acting within and between individual minds? How does social impact affect agents' elaboration of new or contrasting information?

As opinion is still a debated concept within several disciplines, either its conceptualization or formalization are hard tasks. In particular, the actual instruments -e.g. metrics, formalisms does not allow for a tight definition accounting for a) the relationships between opinions and other epistemic representations and b) their dynamics both at social and individual level. In this paper we approach a preliminary *formal* definition of opinions by means of *Time Varying Graphs* [8]- e.g. a new formalism aimed at characterizing dynamically evolving systems as shown in [23, 22].

In section 2, a brief review of the state of the art is provided to introduce the main theories of opinions developed in the field of social psychology and to discuss more recent advances in opinion dynamics. Section 3 is devoted to the description of our model, in which a definition of opinions as specific mental representations and cognitively founded hypotheses about their diffusion and change will be put forward. In section 4 a preliminary formal account of how opinions are generated and how they can change is provided. In section 5 some conclusions are drawn and future directions are suggested.

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2 State of the Art

Social psychologists have devoted much attention to the study of opinions' formation and spreading, but a comprehensive and definite model allowing for an operational and generative account is still missing. Providing a comprehensive review of social psychology literature is beyond the scope of this work, but in this section we will discuss some of the main theories in order to underline how partial is the picture of opinions emerging from these studies.

In general, opinions are treated as synonyms for different mental objects, as beliefs [20], or more frequently, attitudes. Opinions are often conceptualized as attitudes [19], [15], [21] or they are used as interchangeable terms that have in common the fact of being affected by social influence and persuasion [25]. Allport [3] recognizes the difference between attitudes and opinions but he nonetheless considers the measurement of opinions as one way of identifying the strength and value of personal attitudes. An alternative view contrasts the affective content of attitudes with the more cognitive quality of opinions that involve some kind of conscious judgements [12]. Crespi [9] considers individual opinions as "judgemental outcomes of an individual's transactions with the surrounding world" (p.19), emphasizing the interplay between what he calls an attitudinal system and the external world characterized by the presence of other agents and different subjective perceptions. Opinions are the outcomes of a judging process but this does not mean that they are necessarily rational or reasoned, although Crespi recognizes that they need to be consistent with the individual's beliefs, values and affective states. As other authors already pointed out [1], many models of opinion and social influence do not provide careful definitions of what an opinion is and how it is affected by social influence. This happens to be true also for theories of persuasion, like the social impact theory [16], a static theory of how social processes operate at the level of the individual at a given point in time. Part of this theory has been developed usign computational modeling by Nowak, Szamrej and Latan [2]. In their model, individuals change their attitudes as a consequence of other individuals' influence. In parallel with the idea that social influence is proportional to a multiplicative function of the strength, immediacy, and number of sources in a social force field [16], [13] suggest that each attitude within a cognitive structure is jointly determined by the strength, immediacy, and number of linked attitudes as individuals seek harmony, balance, or consistency among them. Although very interesting, this account fails to distinguish between attitudes and beliefs and does not explain how inconsistencies can be resolved. The effect of communication on opinion formation has been addressed by different disciplines from within the social and the computational sciences, as well as complex systems science (for a review on attitude change models, see [1]). One of the first works on this topic has focused on polarization, i.e. the concentration of opinions by means of interaction, as one main effect of the "social influence" [11], whereas the Social Impact Theory' [2] proposes a more dynamic account, in which the amount of influence depends on the distance, number, and strength (i.e., persuasiveness) of influence sources. As stated in ([7]), an important variable, poorly controlled in current studies, is structure topology. Interactions are invariably assumed as either all-to-all or based on a spatial regular location (lattice), while more realistic scenarios are ignored.

Turning our attention to complex systems science, one of the most popular model applied to the aggregation of opinions is the bounded confidence model, presented in [10]. Much like previous studies, in this work agents exchanging information are modeled as likely to adjust their opinions only if the preceding and the received information are close enough to each other. Such aspect is modeled by introducing a real number ϵ, which stands for tolerance or uncertainty ([7]) such that an agent with opinion x interacts only with agents whose opinions is in the interval $]x-\epsilon, x+\epsilon[$.

The model we present in this paper extends the bounded confidence model by providing a cognitively plausible definition of opinion as mental representations and identifying their constitutive elements and their relationships.

2.1 Main Advances

This work aims at outlining a non-reductionist cognitive model of opinions and their dynamics. Differently from the models reviewed above, we first provide a definition of opinions as mental representations presenting specific features that make their revision and updating more or less easy and enduring. Moreover, grounding opinions in the minds allow us to take into account not only direct processes of revision triggered by the comparison with others' different opinions, i.e. social influence, but also revisions based upon changing in other mental representations supporting that opinion.

The computational model introduced in this paper is intended to provide a preliminary unifying framework to define opinions and to characterize their dynamics in an easy but non-reductionist approach. Opinions in several models of opinion dynamics are considered to change according to social influence, we try to outline what is social influence and the way the social network structure affects the agents' opinions.

3 A Cognitive Theory of Opinions

Opinions can be described as configurations of an individual's beliefs, values and feelings that can be conditionally activated. This means that, for instance, starting from my feeling of aversion toward mathematics and as a consequence of having met a rude friend of friends who happened to teach math at school, when asked about my opinion on the time kids should spend in studying mathematics, I can form or, better, activate an opinion according to which the less time they spend the better it is. Opinions stem from the conditional activation of different kinds of mental representations, that can have a propositional content or, as in the case of attitudes and feelings, they can be more evaluative. However, there is a specific feature that distinguishes an opinion from other kinds of mental objects. An opinion is an epistemic representation, thus it is a belief in which the truth-value is deemed to be uncertain. Opinions refer to objects of the external world that can not be told to be either true or false. This impossibility to say whether the content of a representation is true or false is what makes a mental representation an opinion, as opposed to a piece of knowledge, for instance. This basic feature can be paired with the presence of an attitude, i.e. an evaluative component that specifies whether the individual likes or dislikes the topic. In general, attitudes are present when the topic is somehow involving for the subject, so he is positively or negatively inclined toward it.

When this is not the case, we have "factual opinions", like in the following example. If someone is required to say when Mozart died, he can know the correct answer or not, but this is not a moot point. On the contrary, the causes of Mozart's death are debatable because without knowing where he was buried it is impossible to analyze the bones and to ascertain what killed him. This means that we know that Mozart died in 1791 but there are contrasting opinions about the causes of his death, and, even if there exist one true opinion, none can tell which is the truth. On the other hand, when opinions involve also evaluative components or facts, the opinions result from the activation of a pattern of related representations like knowledge, other opinions, but also goals. This view allows us to describe opinions as non-static patterns of relationships in which different representations are linked through a variety of different linkages. This work is meant to address the origin and changing of opinions thanks to these inter-relationships.

An opinion is characterized by the three following features. First, the truth value can not be verified (or it is not relevant). In general, opinions are representations whose truth value can not be assessed through direct experience. The topic of the opinion can not be experienced and then it is impossible to say whether a given object is true or false. If I ask someone about his opinion on the military intervention in Afghanistan, he can not tell me that his opinion, whether positive or negative, is true, because it is not possible to test an alternative state of the world in which the intervention has not taken place and then asses which state was the best. Nonetheless, he can tell me that he has a strong opinion or that he is very confident in it because he has many supporting beliefs (e.g. Talibans' regime had to be fighted, civilians needed the intervention, the world is a safer place after the intervention, etc) and even some goals (for instance, feeling safer) related with that opinion. This is to say that the lack of an assessable truth value is totally independent from the confidence one has in his opinions. We can have strong or weak opinions, but our confidence does not depend on the fact that something is known to be true, given the impossibility to assess its truth-value.

The second feature is the degree of confidence which is a subjective measure of the strength of belief and it expresses the exent to which one's opinion is resistant to change. The degree of confidence depends on the number of supporting representations, and the higher this number the stronger an opinion will be. Castelfranchi, Poggi [6] made a distinction between confidence coming from the source and confidence coming from the degree of compatibility that a given belief has with pre-existing beliefs. It is interesting to notice that representations do not need to be about the same topic or to belong to the same set to form a coherent network. If we take the Afghanistan example, we can easily imagine that a negative opinion about the military intervention could be supported by a general belief about the right of other countries to intervene in internal disputes or by negative evaluations about the US foreign policy, or even by knowledge about the roles played by URSS and US in Afghanistan during the Cold War. These beliefs are not exclusively related to the target opinion and they can have stronger or weaker connections with other opinions. The stronger the confidence in these beliefs and the higher their number, the stronger will be the confidence in that opinion.

Finally, the sharing of an opinion, i.e. the extent to which a given opinion is considered shared, is another crucial feature. The sharing may heavily affect the degree of confidence, making people feel more confident because many other individuals have the same opinion. The sharing is the outcome of a process of social influence, through which agents' opinion are circulated within the social space and they can become more or less shared. This dimension is crucial, but it is also true that it carachterize other social beliefs, like reputation.

It is worth noticing that there are other kinds of beliefs that are really close to opinions but, at a closer investigation, there are some important differences. Reputation can be one of these, because it is shared and it is also carachterized by a varying degree of confidence. But, unlikely opinions, reputation has a truth value because it refers to someone's behaviors or actions that were actually exhibited (or that were reported as such, but we do not want to address here the issue of lying) and reported to other people. Reality matters in reputation, whereas it is much less relevant in opinions, as witnessed also by the fact that reputation does not have to be convincing (i.e. supported by some reasoning or arguments), whereas opinions have.

4 Toward a Formal Definition

4.1 Preliminaries

4.1.1 Time Varying Graphs

The temporal aspects of our opinion model is based on Time-Varying Graphs (TVG) formalism, a generic mathematical framework [8] designed to deal with the temporal dimension of networked data and to express their dynamics from an *interaction-centric* point of view [26].

Consider a set of entities V (or *nodes*), a set of relations E between these entities (*edges*), and an alphabet L accounting for any property such that a relation could have (*label*); that is, $E \subseteq V \times V \times L$. L can contain multi-valued elements.

The relations (interactions) among entities are assumed to take place over a time dimension (continuos or discrete) $\mathcal{T}$ the *lifetime* of the system which is generally a subset of $\mathbb{N}$ (discrete-time systems) or $\mathbb{R}$ (continuous-time systems). The dynamics of the system can subsequently be described by a time-varying graph, or TVG, $\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$, where

- $\rho : E \times \mathcal{T} \to \{0, 1\}$, called *presence function*, indicates whether a given edge or node is available at a given time.
- $\zeta : E \times \mathcal{T} \to \mathbb{T}$, called *latency function*, indicates the time it takes to cross a given edge if starting at a given date (the latency of an edge could vary in time).

4.1.2 The underlying graph

Given a TVG $\mathcal{G} = (V, E, \mathcal{T}, \rho, \zeta)$, the graph $G = (V, E)$ is called *underlying* graph of $\mathcal{G}$. This static graph should be seen as a sort of *footprint* of $\mathcal{G}$, which flattens the time dimension and indicates only the pairs of nodes that have relations at some time in a given time interval $\mathcal{T}$. In most studies and applications, G is assumed to be connected; in general, this is not necessarily the case. Note that the connectivity of $G = (V, E)$ does not imply that $\mathcal{G}$ is connected at a given time instant; in fact, $\mathcal{G}$ could be disconnected at all times. The lack of relationship, with regards to connectivity, between $\mathcal{G}$ and its footprint G is even stronger: the fact that $G = (V, E)$ is connected does not even imply that $\mathcal{G}$ is "connected over time".

4.1.3 Edge-centric evolution

From an edge point of view (relationships within epistemic representations), the evolution derives from variations of the availability. TVG defines the *available dates* of an edge e, noted $\mathcal{I}(e)$, as the union of all dates at which the edge is available, that is, $\mathcal{I}(e) = \{t \in \mathcal{T} : \rho(e, t) = 1\}$. Given a multi-interval of availability $\mathcal{I}(e) = \{[t_1, t_2) \cup [t_3, t_4) ...\}$, the sequence of dates $t_1, t_3, ...$ is called *appearance dates* of e, noted $App(e)$, and the sequence of dates $t_2, t_4, ...$ is called *disappearance dates* of e, noted $Dis(e)$. Finally, the sequence $t_1, t_2, t_3, ...$ is called *characteristic dates* of e, noted $\mathcal{S}_{\mathcal{T}}(e)$.

4.1.4 Graph-centric evolution

From a global standpoint, the evolution of the system can be derived by a sequence of (static) graphs $\mathcal{S}_\mathcal{G} = G_1, G_2..$ where every G_i corresponds to a static *snapshot* of $\mathcal{G}$ such that $e \in E_{G_i} \iff \rho_{[t_i,t_i+1)}(e) = 1$, with two possible meanings for the t_is: either the sequence of t_is is a discretization of time (for example $t_i = i$); or it corresponds to the set of particular dates when topological events occur in the graph, in which case this sequence is equal to $sort(\cup\{\mathcal{S}_\mathcal{T}(e) : e \in E\})$. In the latter case, the sequence is called *characteristic dates* of $\mathcal{G}$, and noted $\mathcal{S}_\mathcal{T}(\mathcal{G})$.

4.2 Modeling Epistemic Representations

An *opinion* is an epistemic representation of a state of the world with respect to a given object p. It is defined on a three dimensional space defined by: a) the *objective truth value* T_o, a *subjective truth value*, namely T_s and a *degree of confidence* d_c with respect to the object p.

More formally we can state that:

Definition 1 *an epistemic representation of a state of the world* $m \in M$ *is a quadruplet* p, T_o, T_s, d_c *defined by a preposition p related to a given object O, and two variable* T_o *and* T_s *defined on* $\mathbb{R}$*. The* $d_c \in \mathbb{R}$ *respectively quantifying the "real" truth value of an information, namely the objective truth value, the perceived truth values, and the degree of confidence, with respect to the preposition p.*

By varying the dimensions of the domain of T_o and T_s, we can define a taxonomy of the epistemic representation of the world that can be summarised as follows:

Definition 2 *An epistemic representation* $m_k = \{p, T_o, T_s, d_c\}$ *is* knowledge *when* $T_o = T_s$.

Definition 3 *An epistemic representation* $m_b = \{p, T_o, T_s, d_c\}$ *is a* belief *when* $0 < T_o < 1 \land 0 \leq T_s \leq 1$.

Definition 4 *An epistemic representation* $m_o = \{p, T_o, T_s, d_c\}$ *is an* ***opinion*** *when* $0 \leq T_o < 1 \land 0 \leq T_s \leq 1$.

4.3 Opinions and Individuals

We can define an epistemic representation graph as a network of epistemic representation immerged in a dynamic network in a given time interval and the links state the correlation among them. Let us consider a set V of mental representation (or nodes), interacting with one another over time. Each *relation* among the mental representation can be formalized by a quadruplet $c = \{u, v, t_1, t_2\}$, where u and v are the involved mental representations (either beliefs, or knowledge or an opinion), t_1 is the time at which the correlation occurs, and t_2 the time at which the relation terminates. A given pair of nodes can naturally be subject to several such interactions over time (and for generality, we allow these interactions to overlap). Given a time interval $\mathcal{T} = [t_a, t_b) \subseteq \mathcal{T}$ (where t_a and t_b may be either two dates, or one date and one infinity, or both infinities), the set $C(\mathcal{T})$ (or simply C) of all interactions occurring during that time interval defines a set of intermittently-available edges $E(\mathcal{T}) \subseteq V \times V$, such that:

$$\begin{aligned} \forall u, v \in V, (u,v) \in E(\mathcal{T}) \\ \iff \exists t' \in [t_a, t_b), (u, v, t_1, t_2) \in C(\mathcal{T}) \ : \ t_1 \leq t' < t_2 \end{aligned} \quad (1)$$

that is, an edge (u, v) exists iff at least one interaction between u and v occurs, or terminates, between t_a and t_b. The intermittent availability of an edge $e = (u, v) \in E(\mathcal{T})$ is described by the *presence function* $\rho : E(\mathcal{T}) \times \mathcal{T} \rightarrow \{0, 1\}$ such that $\forall t \in \mathcal{T}, e \in E(\mathcal{T})$:

$$\rho(e,t) = 1 \iff \exists(u, v, t_1, t_2) \in C : t_1 \leq t < t_2 \quad (2)$$

The triplet $\mathcal{G} = (V, E, \rho)$ is called an *epistemic representation graph*, and the temporal domain $\mathcal{T} = [t_a, t_b)$ of the function ρ, is the *lifetime* of $\mathcal{G}$. We denote by $\mathcal{G}_{[t,t')}$ the *mental representation subgraph* of $\mathcal{G}$ covering the period $[t_a, t_b) \cap [t, t')$

Hence, a sequence of couples $\mathcal{J} = \{(e_1, t_1), (e_2, t_2), ...\}$, with $e_i \in E$ and $t_i \in \mathcal{T}$ for all i, is called a *journey* in $\mathcal{G}$ iff $\{e_1, e_2, ...\}$ is a walk in G and for all i, $\rho(e_i, t_i) = 1$ and $t_{i+1} \geq t_i$. Journeys can be thought of as *paths over time* from a source node to a destination node (if the journey is finite).

Let us denote by $\mathcal{J}^*_\mathcal{G}$ the set of all possible journeys in an epistemic representation system $\mathcal{G}$. We will say that $\mathcal{G}$ *admits* a journey from a node u to a node v, and note $\exists \mathcal{J}_{(u,v)} \in \mathcal{J}^*_\mathcal{G}$, if there exists at least one possible journey from u to v in $\mathcal{G}$.

4.4 Opinion Dynamics and Society

One of the most famous formalisms aimed at describing the process of persuasion is the "Bounded Confidence Model" (BCM) where agents exchanging information are modeled as likely to adjust their opinions only if the preceding and the received information are close enough to each other. Such an aspect is modeled by introducing a real number ϵ , which stands for tolerance or uncertainty such that an agent with opinion x interacts only with agents whose opinions is in the interval]x ϵ , x + ϵ [. Neverthless the wide, massive and cross-disciplinary use of the BCM ([18, 14]) ranging from "viral marketing" to to the Italians' opinions distortion played by controlled mass media ([24, 4, 5, 14]). Such a model does not provide an explanation of the phenomena yielding to the tolerance value, it is just assumed as a static value.

In this work we will outline which are the factors affecting the acceptance or the refuse of one another opinion. In particular, how can we formalize comparison of two or more opinions? Recalling that a mental representation is a preposition with the truth value defined by two variable $T_o, T_s \in \mathbb{R}$ and $d_c \in \mathbb{R}$ respectively quantifying the "real" and the perceived truth value and the degree of confidence with respect to a given object or proposition. And considering that such mental representations are modeled as set of time connected entities of the form $\mathcal{G} = (V, E, \rho)$ we can now provide some definitions aimed at describing the process of persuasion.

Assuming that an epistemic representation system, which is by nature adaptive, when facing with external events, reacts to the stimulus by activating only a subset of its components. For instance, consider the example where an agent x is questioned by an agent y about his opinion on a given target.

What does happen in the x's mental representation system? How can we quantify x's attitudes to change or not is opinions regarding a given matter of fact?

According to our model the epistemic representation system of x, as reaction to the external stimulus posed by the y's question, will perform $journey$ within the elements that in its mind are related with the target of the question and on this base will be able to compare its opinion with the one owned by y.

Definition 5 (relational-)connected component induced by an external event *in* $\mathcal{G}_x$ *is defined as a set of nodes* $V' \subseteq V$ *such that*

$\forall u, v \in V', \exists \mathcal{J}_{(u,v)} \in \mathcal{J}^*_{\mathcal{G}}$. *Then* $\mathcal{G}$ *is said connected if it is itself a connected component* ($V' = V$).

Since all nodes in V' are defined by an objective truth value T and a degree of confidence (perceived truth value) d_g it is obvious that the resistence to an opinion to change is denoted by these values in all the nodes in V'.

5 Conclusions

In this preliminary work we tried to sketch a cognitively grounded dynamic model of opinions, in which we defined these mental representations as carachterized by the presence of three specific features. Differently than psychological theories of opinions that usually provide rich definitions that are too complex to be reduced to measurable variables, we isolated three main constitutive elements that characterize this kind of mental representations. On the other hand, we tried to overcome the reductionist approach of opinion dynamic models, in which the richness of human cognitive processes is substituted by easy-to-compute factors poorly related to actual human behaviors. For this reason, we proposed to apply time-varying-graph to develop a formal model able to account for the way in which opinions are generated and change as a function of the presence and opinions of other agents in the network.

We are perfectly aware of the complexity of this issue and this work represents a preliminary attempt to merge the cognitive complexity of opinions with a rigorous formal approach, but there are many problems that we need to address. First, the cognitive model should be refined and specific hypotheses about opinion revision and diffusion should be put forward. Moreover, the robustness of the formal model will be tested and such a model will be implemented in cognitive multi-agent system in order to explore the parameter space upon which our model has been defined. Our ultimate aim is to build up a simulation environment in which agents endowed with heterogeneous representations of the external world interact and this leads to the creation of new opinions, the disappearing of some of the previous ones and, in general, to different distributions of representations in the population.

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Multilevel and Agent-Based Modelling in the Analysis of Differential School Effectiveness

Mauricio Salgado and **Elio Marchione** [1]

Abstract. Multilevel Models (MLM) have pioneered the analysis of hierarchical data, with two or more levels. Agent-Based Models (ABM) are also used to analyse social phenomena in which there are two or more levels involved. This paper addresses the integration between MLM and ABM. To provide a basis of comparison, we focus on differential school effectiveness analysis, where MLM has been well studied, using data from the *London Educational Authority's Junior Project*. A MLM is fitted and an ABM of pupils' educational attainment using a social network structure is built. We reports the results of both models and compare their performances in terms of predictive power. Although the fitted MLM outperforms the proposed ABM, the latter still offers a reasonable fit and provides a causal mechanisms to explain differences in school performance that is absent in the MLM.

1 Introduction

During the last thirty years education researchers have developed models for judging the comparative performance of schools, in what has been known as *differential school effectiveness* [13, 17]. These variable-based models, which have achieved great sophistication, determine the extent to which schools improve pupils' educational attainment. Among those models, Multilevel Models (MLM) are very popular, since they allow the analysis of data that have a hierarchical structure, with two or more 'levels' (e.g., pupils and schools) [14]. However, despite their sophistication, variable-based models do not provide causal explanations for the observed social phenomenon [12]. Thus, MLM are well-suited to identify those differences, but they do not explain why those differences might emerge in the first place, since they do not uncover the generative mechanisms that bring about those differences. When researchers want to understand why some social phenomenon emerges, agent-based models (ABM) might be the best alternative. ABM is a computational method to experiment with models composed of autonomous agents that interact within an environment [10]. For instance, researchers might use ABM to explain differential school effectiveness by focusing on the dynamic of the social networks that shape and are shaped by pupils' interactions within and outside school. Whilst ABM is explanatory, MLM is a sophisticated way for description and hypotheses testing. Nevertheless, the integration of multivariate analysis, such as MLM, and the modelling of generative mechanisms, such as ABM, is a crucial methodological issue.

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This paper explores that possibility by formalising an ABM to explain differential school effectiveness. It describes an ABM to understand the effects of pupils' interactions in educational attainment using a network structure and a methodological strategy to cope with the integration of MLM and ABM. We begin this paper with a brief account of MLM models in education research (Section 2). Then, we describe the data we are using (Section 3) and we use multilevel modelling to evaluate possible group effects and the extent to which differential school effectiveness is present in the data (Section 4). We present our proposed ABM to explain differential school effectiveness (Section 5), explaining the model entities, interactions and main dynamics. The last part of the paper describes a comparison between both modelling techniques (Section 6) and it finishes with the further work we are going to undertake and some strengths and limitations of our approach (Section 7).

2 Multilevel models in education research

In the context of educational research, MLM were developed to adjust simple comparisons of school mean values by using measures of pupil prior achievement and other variables to take account of selection and other procedures that are associated with pupils' achievement but not related to any effect that the schools themselves may have on achievement [11, 19]. Thus, a simple two-level, variance components, model based on data from a random sample of schools can be written as follows, where subscripts i refers to pupil, and j to the school:

$$\begin{gathered} y_{ij} = \beta_0 + \beta_1 x_{ij} + u_j + e_{ij}, \\ u_j \sim N\left(0, \sigma_u^2\right),\ e_{ij} \sim N\left(0, \sigma_e^2\right) \end{gathered} \tag{1}$$

where y_{ij} and x_{ij} respectively are the response variable and prior achievement, and u_j is an underlying school effect or residual (which is associated with school organization, teaching, etc.). As is usual, this model assumes that e_{ij} and u_j are uncorrelated and also uncorrelated with any explanatory variable—i.e. it assumes that any possible dependences that may result from, for example, school selection mechanisms are accounted for. Posterior estimates $\hat{u}_j$ with associated confidence intervals are typically used to rank schools in so-called 'league tables' or used as 'screening devices' in school improvement programmes.

Model (1) can be elaborated by introducing further covariates such as socio-economic background or peer group characteristics, to make additional adjustments, satisfy the distributional assumptions or investigate interactions. In addition, it is typically found that models such as Model (1) require random coefficients, where, for example, the coefficient of prior achievement varies randomly across schools. In this case, using a more general notation, we have

$$\begin{aligned} y_{ij} &= \beta_{0ij} + \beta_{1j} x_{ij}, \\ \beta_{0ij} &= \beta_0 + u_{0j} + e_{ij}, \\ \beta_{1j} &= \beta_1 + u_{1j}, \\ e_{ij} &\sim N\left(0, \sigma_e^2\right), \end{aligned} \tag{2}$$

$$\begin{pmatrix} u_{0j} \\ u_{1j} \end{pmatrix} \sim N\left(0, \Omega\right), \Omega = \begin{pmatrix} \sigma_{u0}^2 & \\ \sigma_{u01} & \sigma_{u1}^2 \end{pmatrix}$$

The Multilevel Model (2) has also been extended to include further levels of hierarchy, such as education board or authority, and random factors which are not contained within a simple hierarchy, such as area of pupil residence or school attended during a previous phase of education. Such designs are known as 'cross-classification'. In any case, when we use a MLM, we assume that the group level makes a difference that explains the total variance of the dependent variable [9]. Therefore, we need to identify how important the group level differences are (i.e., to identify the importance of the 'school effect'), or the proportion of the total variance accounted for the group level. A convenient summary of this effect is the 'interclass-correlation' coefficient (ICC), given by the formula

$$\rho = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2} \tag{3}$$

The proposed ABM should describe a similar pattern, that is, it should reproduce the school effects or differences in the school effectiveness that are in the data as shown by a pattern of high interclass-correlation. The advantage of complementing MLM with a 'bottom-up' approach lies not only in its power to replicate some previous discoveries, but also in the identified causal mechanisms that might bring about the differences in school effectiveness. In the following section we describe the main components and dynamics of such an ABM.

3 Data

In order to implement a two-level model, we used a subsample from the *The London Education Authority's Junior School Project Data* for pupils' mathematics progress over 3 years from entry to junior school to the end of the third year in junior school [13]. This was a longitudinal study of around 2000 children. Our subsample consists of 887 pupils from 48 schools, with five relevant variables, namely:

- *ID School*, an identification number assigned to each school, from 1 to 48,
- *Social Class*, a dummy variable representing father's occupation, where 'Non Manual Occupation' = 1 and 'Other Occupation' = 0,
- *Gender*, a dummy variable representing pupils' gender, where 'Boy' = 1 and 'Girl' = 0, and
- *Math3* and *Math5*, pupil's score in math tests in year 3 and in year 5 respectively.

This data enable us to perform a two-level model (pupils grouped in schools). We estimated an *unconditional means model* [18], which does not contain any predictors but includes a random intercept variance term for groups, and it is defined as $Y_{ij} = y_{00} + u_{0j} + r_{ij}$, where the dependent variable is a function of a common intercept y_{00} and two error terms: the between-group error term, u_{0j}, and the within-group error term, r_{ij}. This model is useful since we can get two estimates of variance from it: τ_{00} for how much each groups' intercept varies from the overall intercept (y_{00}), and σ^2 for how much each individuals' score differs from the group mean. An analysis of this model showed that the ICC (see Equation (3)) equals to 0.119, so an important portion of the variance (12%) is explained by the pupils' group membership. Further, the overall group mean reliability test [4] of the outcome variable equals 0.67, although several schools have quite low estimates. In fact, just 22 over 48 schools have group mean reliability over 0.7, which is the conventional value to determine whether groups can be reliably differentiated. Finally, we get from our unconditional means model that the intercept variance τ_{00} is significantly different from zero, $\chi^2(3) = 52.3, p < .0001$. Therefore, the analysis shows that fitting a MLM is a sensible decision.

However, given the great heterogeneity among the schools in our subsample, which is particularly salient in relation to the group mean reliability, we decided to perform our analysis and simulations considering just those 22 schools that described high estimates in this test, representing 558 pupils. By doing so, we will be working with data that describes stronger group effects. Both the exploratory nature of our research and the early experimental stage we are facing justify this decision.

4 Fitting a Multilevel Model

The multilevel models used for the analysis of later mathematics outcomes (i.e., year 5) were elaborated to take account of relevant background factors and prior attainment (i.e., year 3). We elaborated different models and compared them in order to evaluate their overall fit. Table (1) shows the results of these comparisons. We fit a base model, *Model 0*, whith no predictors but just random intercepts. *Model 1* considers one predictor, in this case previous attainment, and the intercepts of the groups were allowed to vary randomly . *Model 2* includes to the previous model background factors for each pupil, namely: gender and social class. Finally, *Model 3* was fitted considering both previous attainment and background factors and, additionally, the slopes of previous attainment were allowed to vary randomly across the 22 schools considered. The results shown in Table (1) establish that *Model 3*, which allows random slopes, has a significantly better fit to the data in comparison to the random intercept model (i.e., *Model 2*), $\chi^2(2) = 6.8$, $p = 0.034$.

	df	AIC	BIC	*log* Lik
Models				
Model 0	3	3858.127	3871.257	−1926.064
Model 1	4	3660.438	3677.945	−1826.219
Model 2	6	3659.913	3686.174	−1823.957
Model 3	8	3657.157	3692.170	−1820.578
Tests	χ^2	p-value		
0 vs 1	199.689	< 0.001		
1 vs 2	4.525	0.104		
2 vs 3	6.757	0.034		

Table 1. Comparison of Fitted Models

Figure (1) depicts the slopes of previous attainment in *Math 3* for each of the 22 schools selected for the analysis. From these plots it seems likely that there is some slop variation, something that complements the information presented in Table (1), where the log likelihood results indicate that a model with the random effect for the pupil's previous attainment in Maths is significantly better than the model without these random effects. Therefore, a random slope model is selected for the analysis. This means that, in order to establish whether the schools were differentially effective, previous attainment was allowed to vary randomly at both the pupil and the school levels.

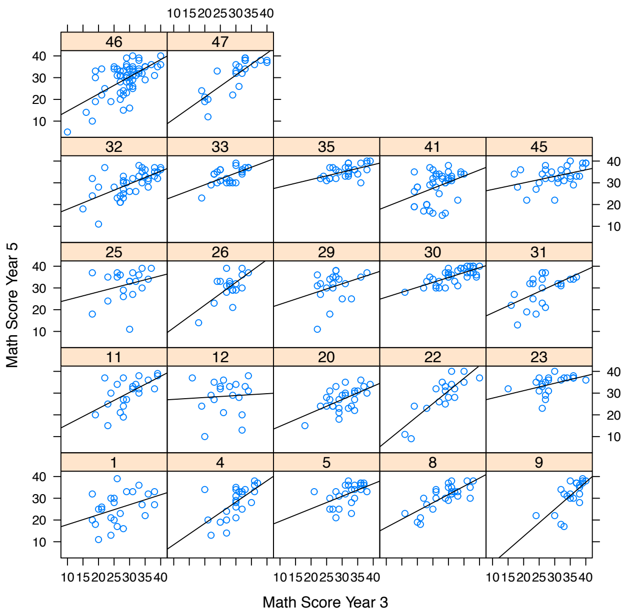

Figure 1. Scatterplots for Each of the 22 School in the Analysis

The results from fitting a model with a random slope for prior maths attainment, controlling by gender and social class, are shown in Table (2) and they can be interpreted following Equation (2). As we can see, the average intercept across all the schools β_0 equals 12.65 (std. error 1.79) and the average slope for Math 3 across the 22 schools β_1 equals 0.6 (std. error 0.05). Both parameters are significant. However, the individual school slopes u_{1j} vary around the average slope with a standard deviation estimated as 0.14. The intercepts of the individual schools u_{0j} also differ, with a standard deviation estimated as 6.04. In addition, there is a negative covariance between intercepts and slopes σ_{u01} estimated as -0.98, suggesting that schools with higher intercepts tend to have lower slopes. Finally, the pupils' individual scores vary around their schools' lines by quantities e_{ij}, the level 1 residuals, whose standard deviation is estimated as 5.17.

The two control variables included in our model, gender and social class, perform differently. In fact, just social class (i.e., '*Nonman*' in Table (2)) is making a contribution to the model, with an estimated regression coefficient of 1.17 (std. error 0.53, $p < 0.05$). Consequently, pupils whose father's occupation is non-manual have an expected advantage of 1.17 points in Math 5 in comparison to those students whose father's occupation is manual. On the other hand, gender (i.e., '*Boy*' in Table (2)) does not contribute to the predictive power of the model, since its regression coefficient is quite low, -0.02 (std. error 0.44) and, consequently, it is not significant.

Parameters (Outcome Variable: Math 5)			
		Random Effects Estimate	
St. Dev. (σ)	Intercept	6.04	
	Math 3	0.14	
	Residual (e_{ij})	5.17	
	Math 3/Intercept	-0.98	
		Fixed Effects	
		Estimate	Std. Error
Coefficients (β_{nj})	Intercept	12.65***	1.79
	Math 3	0.60***	0.05
	Nonman	1.17*	0.53
	Boy	-0.02	0.44

Note. Signif. codes: *** $= p < 0.001$, ** $= p < 0.01$, * $= p < 0.05$.

Table 2. Parameters of Random Slope Model for Previous Attainment

With the information obtained from our MLM, predictions might be carried out for every pupil in one of the 22 schools. Thus, for instance, let us take a boy student from school 32, whose previous attainment in Maths at year 3 was 22, and whose father's occupation is classified as manual. From our MLM we know that the group-intercept for this school $u_{0,32}$ equals 6.7869 and its group-slope for previous attainment $u_{1,32}$ equals -0.1418. These values are incorporated into Equation (2) and we obtain that the predicted value in *Math 5* for this student ≈ 29.5.

5 The Proposed ABM

The ABM we propose addresses the problem of explaining the differences in school effectiveness by taking into account the inputs of knowledge that every student receives from her social environment (i.e., the other individuals with whom the student interacts) in relation to one specific subject they are supposed to learn. Thus, our model considers the relevant social network in which the pupil is embedded.

5.1 Theoretical framework

The importance of taking into account the network in which a pupil is embedded in order to explain her educational attainment is well established in the literature. Since the observational study carried out by Rist [16] in the seventies, educational researchers are aware of the impact the student-teacher relationship might have on pupils' learning. Thus, schools where teachers have higher expectations regarding the future of their students might actually perform better compared to others where teachers have lower expectations [7]. These expectations determine which pupils are defined by the professor as 'fast learners' and which ones as 'slow learners'. By this way, teachers behaviour contribute to the 'self-fulfilling prophecy', that is, pupils that are considered 'slow learners' in advance receive less attention and educational feedback, and consequently, they perform worst compared to pupils who are considered 'fast learners' in advance. Equally important are the pupils' characteristics within the classroom, which effect on children's educational achievement has been well documented. Beckerman and Good [3] studied this element and they discovered that classrooms in which more than a third of the children were 'high-aptitude' students and less than a third of them were 'low-aptitude' performed better than those classrooms in which the opposite relation was true. Their results indicated that both high- and low- aptitude students in the first kind of classroom had greater achievement gains than comparable students in less 'favourable classrooms'. These findings are consistent with the 'peer-effect' hypothesis, something that has been modelled by using Social Network Analysis [6] (however, see [8] for disconfirmatory evidence of peer-effect on educational achievement). Finally, the cultural capital that pupils' families hold has an important effect on students performance [5, 20], being stablished the association between higher social class and lower cultural capital. Hence, previous research in the field allows us to focus on three dimensions that are relevant to explain school differential effectiveness: (a) educational feedback or training in the subject pupils receive; (b) pupil-pupil interactions and (c) pupil's cultural capital. These three social dimensions of education determine the elements we aim to model.

5.2 ABM description

The ABM was designed upon two basic assumptions. The first assumption deals with the way in which pupils' learning of one specific topic evolves over time. It seems reasonable to assume that this learning can be modelled as a logarithmic function of the educational feedback or training received in the subject. Thus, following a logarithmic model, learning growth as a function of number of trainings describes an initial period of rapid increase, followed by a period where the growth in learning slows. Some specific contexts seems to validate this assumption empirically (see, for instance, [2]).

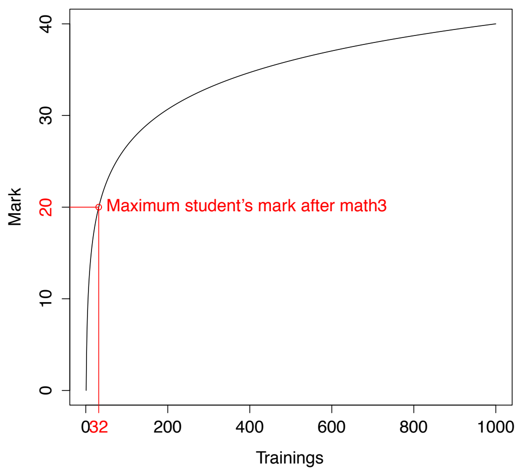

Figure 2. Simulated Pupils' Learning Curve

Therefore, in order to model pupils' learning in maths over time (that is, from year 3 to year 5), we define a students' learning curve. Firstly, we assume that learning maths is a continuum process starting at training 0 and ending when the knowledge of maths is measured in year 5 (or *Math 5*). We arbitrarily define 1,000 as the number of trainings for the entire learning process. This operationalises the teacher-pupil contact time throughout all the learning process. Figure (2) shows the students' learning curve employed in the proposed ABM. Simulated students' marks are, therefore, worked out as a function of the number of trainings they have undertaken. The relation between marks and trainings is a logarithmic function defined between 0 and 1,000 that returns values between 0 and 40. We also assume that when the test *Math 3* was applied, students have learned half of the topics they were supposed to learn on the subject. Further, since both *Math 3* and *Math 5* range between 0 and 40, we transform *Math 3* by dividing it by 2. Secondly, we assume that the number of trainings students undertake depends on the socialisation processes within their schools. By socialisation we mean all those practices and rules that eventually generate stable groups of students. A group is stable when its members do not want to leave, that is, they are 'happy' as members of the specific group. Let g_k be a stable group in a school j and s_{ik} a student in such a group. Let $math3_k$ be the average of *Math 3* scores of group g_k, then the amount of trainings that the students in group k agree to undertake is given by the following equation:

$$t_k = \left(e^{2 \cdot math3_k}\right)^{\frac{1}{5.790593}} \tag{4}$$

Then, the simulated student's score $simMath5_{ik}$ is shown in Equation (5), where $t_{ik} = t_k + t_{math3,i}$ and $t_{math3,i}$ is the number of trainings the pupil had had when her attainment was measured as *Math 3*.

$$simMath5_{ik} = log\left(t_{ik}^{5.790593}\right) \tag{5}$$

The second assumption is related to the group formation mechanisms. We propose a refinement of Resnick and Wilensky's model [15]. There is an initial number of spots where students can hang out at. Students staying at the same spot conform a group. Following the specialised literature, we assume that group formation rules is a permanent tradeoff between individual characteristics and institutional factors [1]. Thus, in our ABM pupils' tolerances towards their schoolmates vary across schools. These tolerances define, in turn, students' comfort levels within a group. If they are in a group that has, for example, a higher percentage of people of the opposite sex than school's tolerance, then they are considered 'uncomfortable", and they leave that group to the next spot. Movement continues until everyone at the school is "comfortable" with their group. The final number of groups might be smaller than the number of spots. Taking into account the available data (see Section (3)), we define three tolerance levels: *Educational tolerance*, that reflects the students tolerance of having others with different attainments in *Math 3*; *Gender tolerance* indicates the students tolerance for people of the opposite sex; and *Social class tolerance*, the pupils tolerance for different social class. If just one of these these three tolerances were not meet, the pupil will leave the group. Tolerance levels range between 0 and 1 and corresponds to the proportion of similar pupils within each group. Figure (3) shows the students network at the end of a simulation for school 32. Male and female pupils are coloured blue and pink respectively; rounded and squared shaped nodes represent low and high social class respectively; and previous attainment in *Math 3* is labelled on students' icons. In this scenario education, gender and class tolerances are 0.9, 0.3 and 0.9 respectively. As we can see, there are 39 students in school 32 and 15 groups.

5.3 Experimental set

We performed a series of experiments with our ABM. The objective of these experiments was to find a set of tolerance levels for each

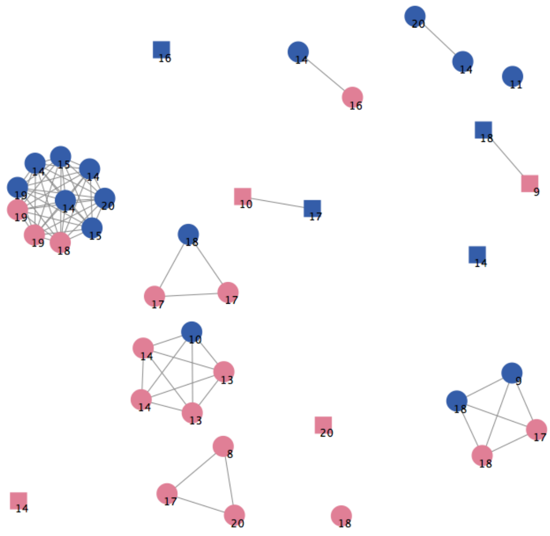

Figure 3. Simulated Students Social Network in School # 32

School Id	Num. Pupils	MLM (d_j)	ABM (d_j)	Final Groups	Tolerance Levels		
					Edu.	Gender	Soc. Class
1	25	2.88	3.36	13	90%	50%	30%
4	24	2.26	3.12	12	90%	90%	50%
5	25	1.53	2.26	12	90%	70%	90%
8	26	1.41	2.82	12	90%	70%	30%
9	21	1.67	2.91	12	90%	70%	30%
11	22	2.21	3.10	12	90%	30%	70%
12	19	3.03	3.55	12	90%	50%	30%
20	28	1.60	2.62	12	90%	30%	70%
22	18	2.18	3.63	10	90%	30%	70%
23	21	1.43	3.19	12	90%	90%	50%
25	20	2.60	3.50	11	90%	30%	50%
26	19	1.85	2.79	12	90%	70%	50%
29	20	2.30	3.36	12	90%	70%	30%
30	35	1.03	2.56	14	70%	90%	70%
31	22	2.30	3.60	12	90%	70%	50%
32	39	1.72	2.71	15	90%	30%	90%
33	25	1.22	3.04	12	90%	30%	90%
35	27	1.01	2.44	13	90%	70%	30%
41	38	2.46	3.25	16	90%	30%	70%
45	30	1.58	2.62	12	90%	30%	70%
46	62	2.24	2.96	15	90%	90%	70%
47	22	1.85	3.61	12	90%	50%	90%

Table 3. Experimental Results

school that minimises the differences between the data and the simulations results. Thus, let d_j be such a difference for school j. Then,

$$d_j = \sum_{i=1}^{n} |math5_i - simMath5_i| / 2 \qquad (6)$$

where $math5_i$ and $simMath5_i$ are the score in *Math 5* of student i obtained from the real data and from the simulations respectively. In the example shown in Figure (3), $d_{32} = 2.231$, which means that the simulated score in *Math 5* differs, in average, form the data in ± 2.231 units. In order to explore the parameter space of the model, we run $126,720$ simulations. The latter number of simulation represents all the possible combination of the three tolerance levels varying among 0.3, 0.5, 0.7 and 0.9 and the number of spots varying among 15, 20 and 25 across the 22 schools considered in this study. In order to have more robust results, we run each setting for 30 times and then took the average of d_j as the aggregate outcome.

6 Integrating MLM and ABM

Table (3) shows the main results obtained from our experimental set. There, we present the average distance (in the same units as the real data) between the predicted scores and the real scores in *Math 5* for both the multilevel model ('MLM (d_j)') and the simulation ('ABM (d_j)') respectively. The results are grouped according to the 22 schools we included in our study. As well, in this table we show the number of groups ('Final Groups') in which all the pupils were happy with their group membership, given the values in the 'Tolerance Levels' for education, gender and social class (see the last three columns of Table (3). Recall that these three last variables were set as simulation parameters, and the specific values presented in the table correspond to those combinations at the school level that minimise the distance between the simulated and the real data scores in *Math 5*. Some remarks might be established.

Firstly, by comparing the average distances between the two models, we see that the predictions MLM outperform the predictions of the ABM, so the former is more accurate. However, the distances of the ABM are not high either; in fact, the overall distance equals 3.04 in a scale of 40 points. Thus, the proposed ABM, despite its simplicity, offers a reasonable fit with the data. Secondly, the simulation results suggest a high educational tolerance, since most of values equal 90% (except from school 30, in which the tolerance level equals 70%). On the other hand, the tolerance levels of social class and gender vary across the schools. Therefore, the group formation mechanism in our simulation seems to be ruled by the variables social class and gender, and previous attainment in maths does not constitute a variable that discriminates between groups. Thirdly, the hypothesised mechanism that bring about the differences in school effectiveness, based on social interactions among pupils and group formation according to tolerance levels defined at the school level, seems to be justified. Actually, the simulation results indicate that the mechanism of group formation helps to minimise the distance between the predicted and the real scores, allowing a better fit with the data. For instance, when we compare the number of groups with the number of pupils, we can see that in general we have less groups than students in each school (for a graphical example, see Figure 3). If the numbers of groups made no difference in the simulation, then the number of groups and the number of pupils would tend to be similar (at least in those schools with number of pupils ≤ 25). This is clearly not the case. Therefore, the pupils' social networks seem to be important to explain the differential effectiveness among schools.

7 Concluding Remarks

In this paper we have presented and integrated the results of two models to address differential school effectiveness. The first one is a MLM, where the hierarchical nature of educational processes is considered. The second one is an ABM, where the social mechanisms that might generate school effects in pupil attainments are formalised and explored. We found that MLM provides reasonably accurate prediction, whereby ABM highlights likely differences across schools that might affect pupils' learning performances. This is a promising study that will be further developed. More sophisticated ABM will be designed to produce prediction as accurate as MLM ones. Furthermore, data coming from the ABM will be fed into the MLM model until the former produces results similar to those in the real data. All in all, integrating a social mechanism based approach of educational phenomena with a hierarchical understanding of these process will be more and more reliable and accurate.

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